THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME LXI MARCH 1925 NUMBER 2

OBSERVATIONS OF THE TOTAL SOLAR ECLIPSE OF SEPTEMBER 10, 1923, BY THE SPROUL OBSERVATORY

By JOHN A. MILLER AND ROSS W. MARRIOTT

ABSTRACT

The paper contains a discussion of the photographs made at Yerbanis, Mexico, by the Sproul Observatory Expedition during the solar eclipse of September 10, 1923. The photographs were most successful. Two special features command attention.

1. *The "disturbed region" of the corona near the southwest limb of the sun.* The ensemble of the complex details of the corona there is almost certainly shown to be immediately above a region of the sun containing sun-spots, photographed by the Mount Wilson observers on September 8 and 9. The corona there contains three series of arches. Measures made by Professor Marriott show that the material in these arches is receding from the sun's center with a velocity of 4.5 miles per second. Above these arches is a nondescript cloud form containing many condensations; these were receding radially from the sun, and also drifting toward the equator. The average velocity of these condensations is about 28 miles per second.

2. In addition to the usual polar rays, *there is at each pole of the sun a sheet of light.* The one around the south pole is very conspicuous, the one around the north pole breaks up into fine streamers which curve very abruptly at their extremity. This is the behavior of streamers formed according to the mechanical theory of the corona, discussed by Miller in *Astrophysical Journal*, **30**, and indicates that these sheets are groups of streamers originating in comparatively low solar latitudes. Measures show that they are rotating with the sun.

Professor H. D. Curtis, with an objective grating, secured the $H\alpha$ line on a "flash" spectrum. $H\alpha$ shows very strongly in an arc of 56°. There is nothing comparable with it in this region, and it seems certain there is no very strong radiation between $H\alpha$ and λ 8500 in the solar "flash" spectrum.

The observation site of the Sproul Observatory Eclipse Expedition was located on the Hacienda-Catalina near Yerbanis (Durango), Mexico, a small station on the National Railroad of Mexico, about

60 miles from the city of Durango. The owners of the Hacienda-Catalina most graciously and generously gave us every necessary privilege and extended to us a welcome in every way characteristic of Latin-American courtesy. The Mexican government, ably represented by Joaquin Gallo, director of the National Observatory of Mexico, extended to all visiting astronomers the most thoughtful courtesy. All who visited the city of Mexico were the guests of the republic. All instruments for observation and the necessary accommodations for living were admitted duty free. The absence of the usual customs annoyances was very delightful. One of the Mexican observing stations under the direction and personal supervision of Director Gallo was situated a short distance from our station. We became indebted to him and his staff for numberless courtesies. He put at our disposal not only his personal services in many instances, but loaned us at various times small instruments and books. His Excellency, Señor Castro, governor of Durango, manifested the keenest interest in the work of the expedition. By letter and by messenger he offered any assistance that was in his power to give. He sent each day an emissary to the camp to inquire if there was anything that he could do which had been left undone. The government stationed a company of soldiers in Yerbanis, and on the day of the eclipse a cordon of soldiers prevented visitors from reaching the camp until totality was over. It is a pleasure to acknowledge our debt of gratitude.

We desired to locate as far east as possible in order to determine by comparison of our photographs with those made on the Pacific Coast, whether or not there was motion in any part of the corona during the eclipse. Professor H. H. Kimball, of the United States Weather Bureau, kindly made a study of the records of the Weather Bureau of the republic of Mexico. From this study, supplemented by the report of the Eclipse Committee of the American Astronomical Society, and by data gathered by correspondence, we chose a station as far east as weather records seemed to justify. As it turned out, we could with advantage have gone farther east than we did.

The longitude of our station was $6^h\,55^m\,22^s.0$ and its latitude was $24°\,44'\,00''$. Yerbanis is almost at the crest of a water shed, has an altitude of 6250 feet, is in an arid region, and for four years

prior to the eclipse almost no rain had fallen there. This year, however, seemed to be exceptional, and we encountered a great many clouds and considerable rain. The last ten days before the eclipse were cloudy most of the day and practically all of the night, and a great deal of rain fell. The eclipse occurred on Monday, September 10; on the preceding Friday and Saturday it rained all day and all night in deadly earnest. On Sunday, the rain ceased and the clouds cleared away long enough for us to see the sun come down the great tube of the 65-foot camera at eclipse time. It then clouded over and remained so until about noon on Monday. From 9:00 A.M. September 10, it rained in torrents for an hour; at 10:00, there was a stream of water half an inch deep flowing over every square inch of the camp; the instruments were drenched and so were our spirits, but at noon the clouds broke away, and when the first contact came it was almost clear in the region of the sun. At the beginning of totality and again at the end, there were light clouds floating over the region of the sun, but at mid-totality it was practically clear and continued so the remainder of the afternoon. It clouded the night after the eclipse was over and was cloudy until after we left there on September 14.

The expedition was financed by the Rubel Foundation and other friends of Swarthmore College. It was rather generously, though judiciously, supplied with funds, and we embrace this opportunity to express the appreciation of the entire staff to those who made the expedition possible.

The personnel of the observing party included: the writers; Director H. D. Curtis, of the Allegheny Observatory; Professor Winthrop R. Wright, Mrs. Miller, and Mrs. Marriott, of Swarthmore; Professor Dinsmore Alter, of the University of Kansas; Professor Long, of Franklin and Marshall College; Adrian C. Rubel and Wilson M. Powell, Jr., of New York, students from Harvard University; Earl L. Williams, George B. Clothier, Bevan Sharples, students from Swarthmore College; and Max B. Miller, Jr., of New York. Mrs. Miller and Mrs. Marriott were two indispensable members of the party, and Alice Elizabeth, the four-year-old daughter of Professor and Mrs. Marriott, was its mascot. Charles Charlton, the representative sent by the Pathé News to make motion

pictures of the eclipse and of the operations involved in the erection, adjustment, and manning of the instruments, was with the party from the beginning.

We set for ourselves the following schedule of observations: (1) photographs of the corona; (2) spectroscopic observations; (3)

d c b a

FIG. 1.—Sproul Observatory Eclipse Expedition. (*a*) The mounting of the 65-foot camera; (*b*) The twin Einstein cameras; (*c*) The Noble-Poor camera; (*d*) The wooden polar axis carrying two spectrographs, one interferometer, three cameras, and one movie camera.

photographs for testing the Einstein Theory; (4) records of the shadow bands; (5) motion pictures of the eclipse.

1. *Photographs of the corona.*—We made thirty-two direct photographs of the corona. Eight of these were obtained by a triple Brashear lens generously loaned the expedition by the United States Naval Observatory, for which it was made. This excellent lens has an aperture of $7\frac{3}{4}$ inches (19.7 cm), and a focal length of 65 feet (19.81 m). The mounting of this camera differs only in minor details from that devised by Professor Schaerberle for use in Chile

in 1893. The lens was supported by a tower nearly 50 feet high, which was surrounded by another tower to protect the support of the lens from the wind. The outer tower also carried the upper end of the 65-foot tube, which was inclined 46° to the horizon. The lower end terminated in a darkroom in which the plates were exposed. The entire structure including the covering of the outer tower and the tube was of wood. The mounting is shown in Figure 1*a*. The camera was focused on the stars.

Fig. 2.—One of the smaller polar axes

The plates in the focus of the lens were placed on a carriage moving so as to counteract the relative motion of the sun and the camera. Each plate was placed in a separate wooden box. A few minutes before totality the lid of each box was removed and the box covered with a heavy cardboard. Professor Marriott and Wilson M. Powell, Jr., made the exposures following the program as indicated in Table I. Professor Marriott placed the boxes on the moving carriage, took off the cardboard, moving it off the plate in a prearranged direction. Mr. Powell replaced the cardboard, moving it in the same direction that Professor Marriott had, so that all parts of the plate received the same exposure. Not a single hitch occurred in carrying out this program. All plates were backed the night before

the eclipse with a backing of burnt sienna and granulated sugar. They were developed in metol hydroquinone. The weather was so damp that we experienced considerable difficulty in drying the plates.

By prearrangement, Professor Marriott, in the darkroom, called the word "go" at the instant that totality began. He based his judgment upon the image at the focus of the 65-foot camera. George B. Clothier counted "one" at the end of the first completed second after the word "go," and then counted aloud all the following seconds until 30 seconds after totality was over. The counts in Table I are the counts given by Mr. Clothier and almost exactly represent the length of time after totality had begun. The photographs obtained with this camera are excellent, and seem not to have suffered at all from the clouds.

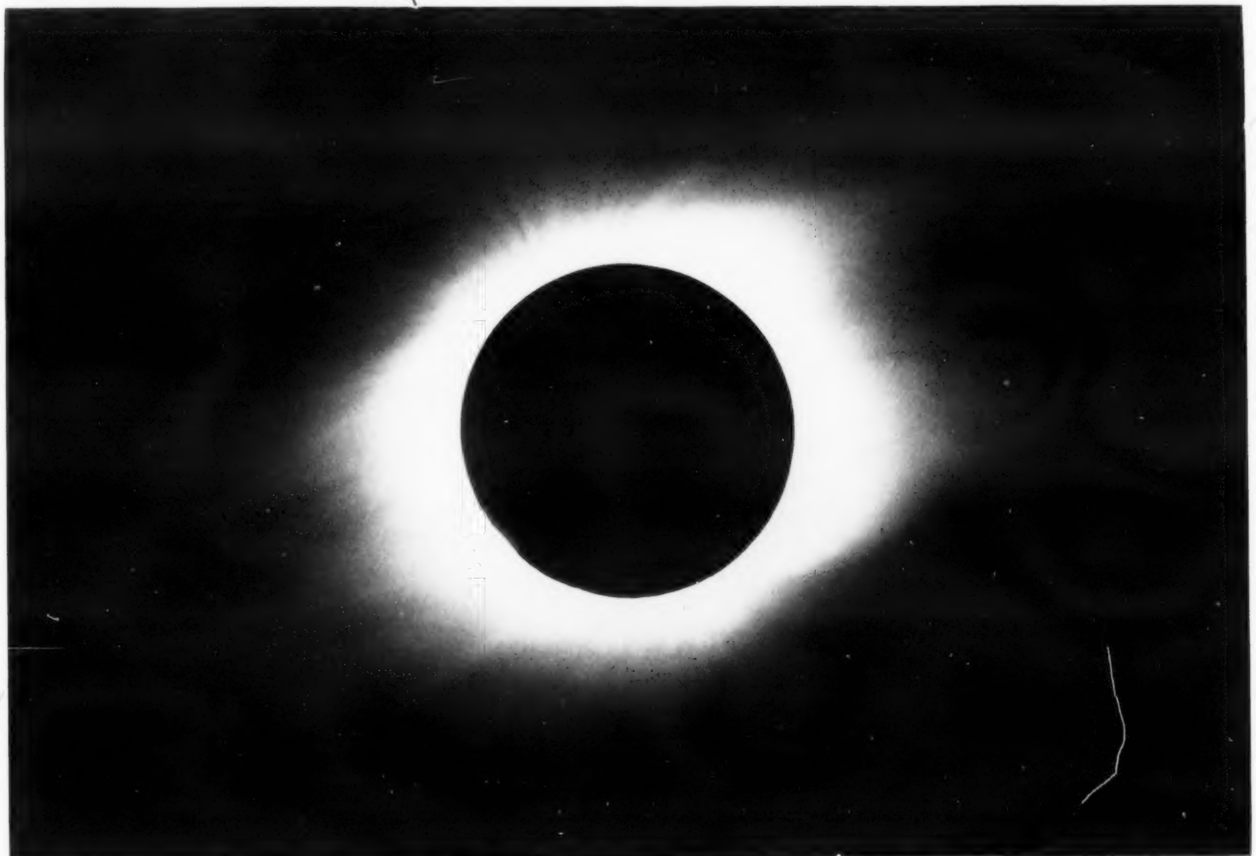
Six very good pictures of the corona were obtained on the six plates exposed in the Einstein cameras. The lenses are 15 feet (4.57 m) focal length. They are discussed below. Mrs. Marriott, in charge of a camera of focal length 104 inches (2.64 m), exposed seven plates, for 2, 4, 8, 16, 28, 65, and 5 seconds, respectively, on Seed 23 plates. Bevan Sharples, in charge of a camera of focal length 61 inches (1.55 m), exposed four plates, for 2, 21, 56, and 64 seconds. Max B. Miller, Jr., in charge of a camera of focal length 38 inches (0.965 m), exposed seven plates, for 2, 4, 8, 16, 28, 65, and 5 seconds. The plates used were Seed 23 and lantern-slide emulsion. Although the photographic scales of these plates are small, they show well-defined detail and considerable extension of the corona. On the third plate exposed in the 38-inch camera two streamers could be traced three and one-half diameters from the limb of the sun. This is the longest coronal extension found on any of the plates. It is hardly necessary to add that the longest exposures were greatly overexposed. The exposure times for the 65-foot camera and the Einstein cameras are given in Table I.

2. *The corona.*—The general type of the corona is characteristic of that of the sun-spot minimum. The photographs made with the 65-foot camera showed that the corona was a little less complex than usual, particularly on the east side of the sun where long, graceful, sweeping streamers can be traced to a distance of three-

PLATE II

N

W



Eq.

Axis

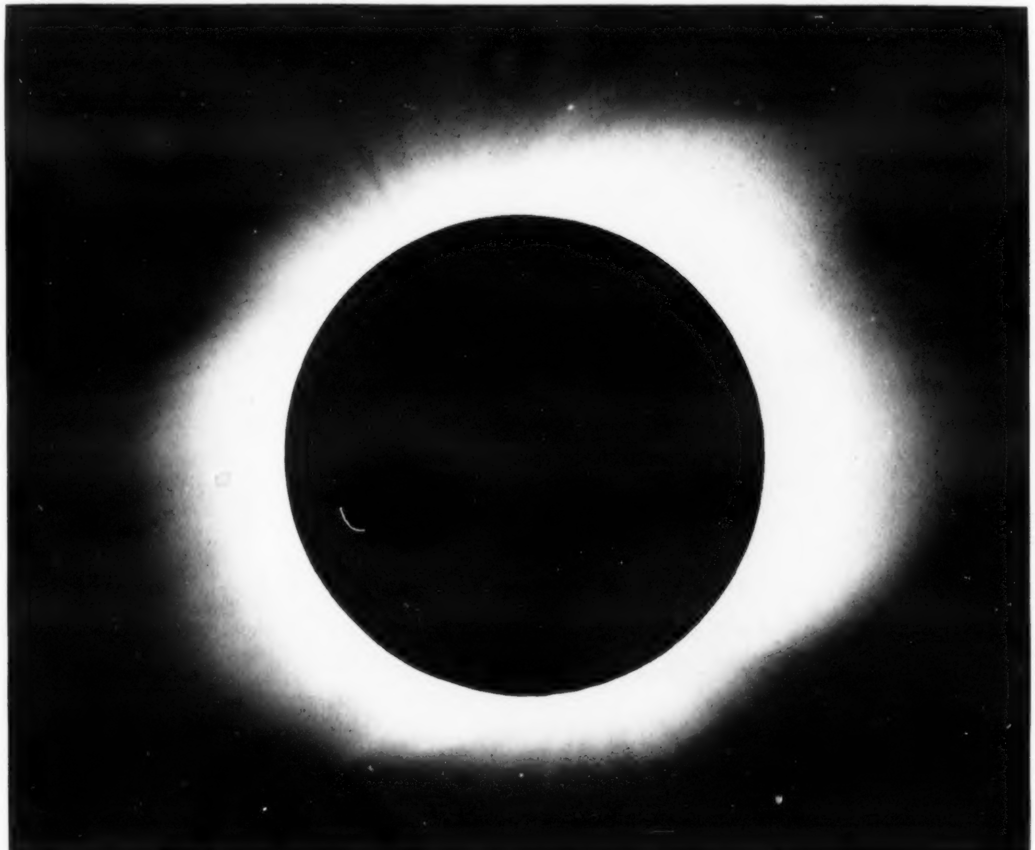
PHOTOGRAPHED BY SPROUL OBSERVATORY EXPEDITION, YERBANIS, MEXICO, SEPTEMBER 10, 1923

Aperture of lens $6\frac{3}{4}$ inches. Focal length 15 feet. Exposure 56 seconds from the beginning of the eclipse

PLATE III

PLATE III

North



Corona, Made with 65-Foot Camera

fourths of a solar diameter from the edge of the sun. There were three prominences on the eastern limb of the sun which do not, so far as appearance goes, seem to have affected the shape of the long streamers. The corona is very much more complex on the southwestern limb of the sun. In this region there is a large eruptive prominence, and a few degrees north of it a series of arches somewhat resembling those conspicuous in the corona of 1918. They were smaller than those of 1918 and less symmetrical. A longer discussion appears below. At a distance of about 10′ above these arches, there is a nondescript, cloudlike form. The corona in this region seems to have been subjected to unusual forces. It seems entirely

TABLE I

Camera	No. of Plate	Time of Exposure	Length of Exposure	Plate
65 feet.......	1	Go − 5^s	5 sec.	Seed 23
	2	12^s − 21	9	23
	3	28 − 53	25	23
	4	60 − 115	55	30
	5	122 − 155	33	30
	6	162 − 171	9	23
	7	175 − 180	5	23
	8	185 − 187½	2½	30
15 feet.......	0, 3	Go − 55^s	55	30
	1, 2	133 − 187	54	30
	5	1½ − 63	61½	30
	4	123 − 178	55	30

probable that the spot group photographed at Mount Wilson Observatory on September 8 and 9, the arches, and this nondescript cloud, are intimately associated. The motion of this cloud is discussed below.

The polar streamers are very long and graceful, resembling those usual at sun-spot minimum. There is an additional phenomenon visible in the region of the poles that neither of the writers has previously seen. At the south pole, from 13° west of the projection of the solar axis to 9° east of it and about one-half the radius of the sun in height, there is what appears to be a sheet of light, practically of uniform brightness throughout. The termination at the greatest distance above the sun is sharp and definite. It appears

on every one of the longer exposures and also on the plates made by Professor Douglas, at Puerto Libertad, 24 minutes before ours were made. The sheet seems to be in front of the polar streamers proper. There is a corresponding sheet at the north pole extending from 0° on the east side of the solar axis (projected) to 25° on the west side. It is about one-third of a solar radius in height. The northern sheet is much more broken up into details, and it is perfectly evident that it is composed of a great number of fine streamers, which, at their greatest distance above the sun, curve very abruptly. This phenomenon is discussed later. It is hardly seen on the engravings.

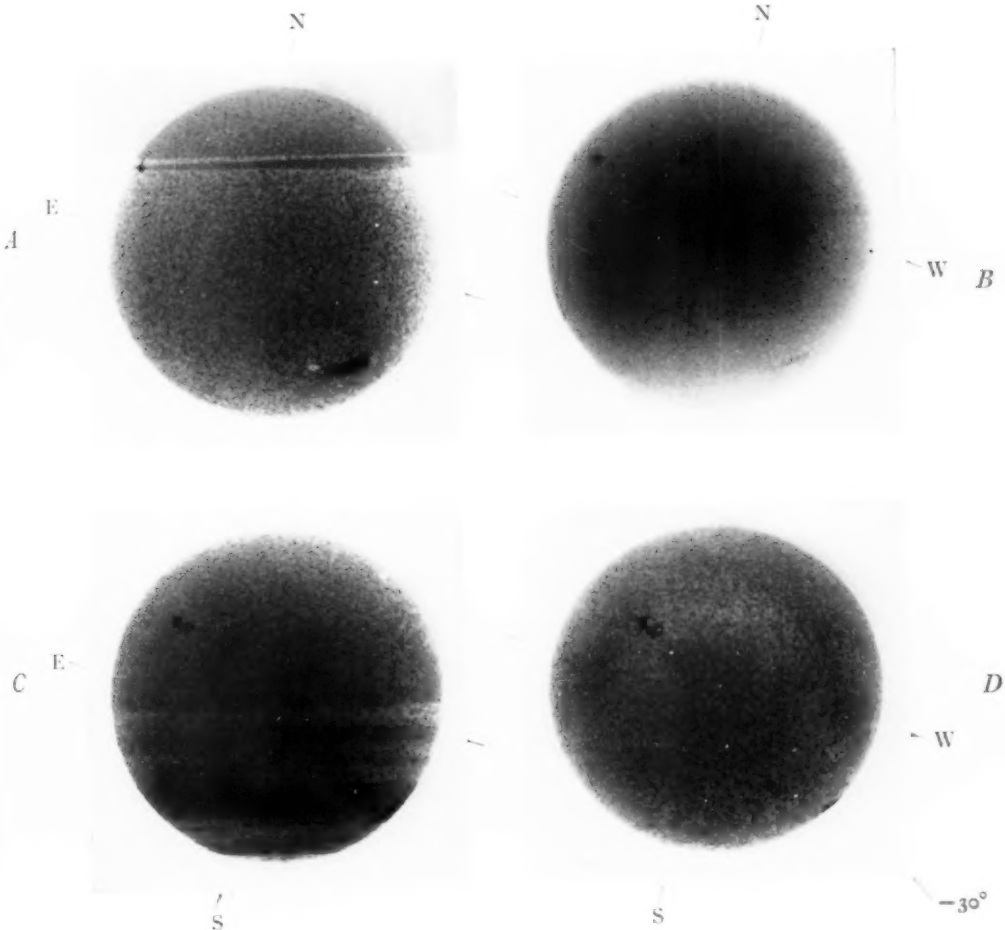
3. *A disturbed region on the southwest limb of the sun.*—We shall now discuss the various phenomena referred to above, beginning with the region on the southwest limb of the sun. It extends from 30° to 37° south of the solar equator. The coronal structure there, instead of being simple as in the remainder of the corona, is very complex, and seems to have been violently disturbed, the outstanding features being a series of interlacing arches. One of these arches, being brighter than the others, is well defined on all the plates. Its greatest height is about 3′ above the solar surface, and the spread of its base subtends an angle of 6°.5 at the center of the sun. On the Sproul negatives, there is a series of six additional fainter arches similar in shape and above this one, the last one of the series being 8′.5 above the solar surface. There is another series of arches similar in many respects to this one south of the first series, and still another north of it. The upper arches may have expanded slightly laterally, but not much so far as we could tell by measures. The base had no greater spread on the Sproul plates than on the Douglas plates. If one drew from the center of the base a line joining the vertices of a given series of arches surrounding each other, this line would curve toward the equator.

The staff of Mount Wilson Observatory very courteously sent us drawings of the K_2 images of the sun made there on September 8, 9, and 10, 1923 (see Plate V). The K_2 image of September 8, $4^h 48^m$ G.M.T., shows a large disturbed region a little in front of the southwest limb of the sun. Three pairs of sun-spots are present in this region. The image made on September 9, $5^h 37^m$ G.M.T., contains the same general features, except that the sun-spots of the

PLATE IV

SECTION OF THE CORONA SHOWING A DISTURBED REGION ON THE SOUTHWEST LIMB OF THE SUN

PLATE V



K_2 SPECTROHELIOGRAMS TAKEN AT MOUNT WILSON OBSERVATORY

(*A*) 1923, September 7, 7.60; (*B*) September 8, 8:46; (*C*) September 9, 9:39; (*D*) September 10, 8:62. (A line from the lower right-hand corner of block to center of sun is −30° heliocentric latitude.)

previous day seem to have collected into two larger sun-spots. The distance between these spots subtends an angle of about 7° at the center of the sun. The K_2 image of September 10 shows this region on the limb of the sun with the sun-spots exactly at the base of the well-defined arches shown on the eclipse plates.

These arches differ in some respects from those so numerous on the eclipse plates of 1918. The arches on the plates in the 1918 eclipse appeared over eruptive prominences. There were no such prominences immediately below the disturbed region on the southwest limb of the sun shown in the K_2 images of September 8 or 9 made at Mount Wilson, or on the eclipse plates made on September 10. There is also another difference. The shapes of the 1918 arches were ogival; those on the 1923 plates were not. The 1918 arches were very much brighter at their bases than they were at their vertices. This was not true of the arches of 1923. Their appearance makes it seem probable, to us almost certain, that the arches are formed from coronal matter and not from prominence material.

4. *The motion in the corona*.—We attempted to ascertain whether or not this material had moved during the eclipse. Professor A. E. Douglas, whose eclipse station was located at Puerto Libertad, Sonora, Mexico, Professor Frank P. Brackett, whose station was located at the "Isthmus" on Santa Catalina Island, and Mr. James Worthington, located at Lompoc, California, most courteously and generously supplied us with glass positives made from their eclipse negatives. These we compared with our own negatives and also with glass positives made from our negatives. We finally decided to measure our own plates and those of Professor Douglas, using the others for checking details of structure. The plates of Professor Douglas were made with a lens of focal length about 40 feet (about 12 m), which was fed by a coelostat. The eclipse took place at Puerto Libertad about 24 minutes before it occurred at Yerbanis.

The center of the moon's image on the glass plate was determined geometrically. The computed differences between the right ascensions and declinations for the sun and moon were then used to determine the center of the sun relative to the center of the moon's image. The center of the sun was plotted on the plate and used as the origin for a system of polar co-ordinates of a series of points on

the arches located as follows. The initial line was determined from an invariant and distinctive marking near the north pole of the sun, and checked against our known east and west line. A series of angles was laid off from the initial line, differing from each other by 1°, and the radial distance measured from the center of the sun to the corresponding point on the arch. Each radial distance is the mean of five independent measures. The distance on the Douglas

TABLE II

Vectorial Angle	Arch I. Average of January and March Measures			Vectorial Angle	Arch I. Average of June Measures		
	Sproul	Douglas	S−D		Sproul	Douglas	S−D
4°	3.78	3.72	0.06	4°	3.74	3.68	0.06
5	3.86	3.80	.06	5	3.84	3.78	.06
6	3.91	3.86	.05	6	3.90	3.84	.06
7	3.89	3.85	.04	7	3.90	3.85	.05
8	3.85	3.79	.06	8	3.87	3.82	.05
9	3.81	3.76	.05	9	3.82	3.77	.05
10	3.75	3.69	0.06	10	3.75	3.70	0.05

Second Arch

Vectorial Angle	Sproul	Douglas	S−D
5°	4.05	3.95	0.10
6	4.06	3.98	.08
7	4.03	4.00	.03
8	3.94	3.94	.00
9	3.89	3.86	0.03

plates was reduced to the scale of the Sproul plates by multiplying them by the factor $1.0017\,\frac{Ds}{Dd}$, where Ds and Dd are the measured diameters of the moon on the Sproul and Douglas plates respectively, and where the factor 1.0017 is calculated from the hour angles, zenith distances, and the distances of the moon from the center of the earth at the two stations at the time the plates were made. The brightest and best-defined arch referred to above we called arch I. Measures were made on January 2, 4, and 17, on March 24 and 29, and again on June 14. Table II shows the average of these measures. The first column contains the vectorial angle, the second and third columns contain the radial distances from the

center of the sun to the corresponding points on the arches as measured in inches on the Sproul and Douglas plates respectively. The fourth column contains the difference of these radial distances. All these distances are reduced to the scale of the Sproul plates.

The individual results are very consistent, not a single measure giving a negative sign for Sproul *minus* Douglas. The largest residual was 0.04 inch and the average of all residuals was 0.013 inch. There may have been an error in finding the center of the sun. However, the center of the sun and the initial line were determined anew on each plate for each set of measurements. A second error may have been made because the instruments are of different focal length, or because the plates were not of the same photographic density, matters that should receive serious consideration. These possible sources of error were discussed by Miller[1] in 1920. To obviate them as far as possible, the streamers were measured on the Sproul plates on negatives that had been given exposure-times of 5 seconds and 9 seconds. They were also measured on contact positives. There were no systematic differences in these measures. Professor W. A. Cogshall, of Indiana University, mounted a lens near Ensenada, Lower California, that had almost exactly the same focal length as our lens. Before going to the eclipse, we had agreed with Professor Cogshall on a program so that the plates for comparison should have equivalent exposures with lenses of nearly the same focal length. Unfortunately, it was cloudy at Cogshall's station. We had hoped by this arrangement to avoid any semblance of criticism touching the sources of error above mentioned. It is our belief that the foregoing measures show that the arches have changed in this 24-minute interval and that they are going outward from the sun, at the rate of about 4.5 miles (7 km) per second.

Almost directly over these arches, there is an immense cloud of coronal matter. It is a conspicuous marking but so devoid of definite form as to baffle description. It extends from 10′ to 12′ above the sun's limb. Assuming that it is exactly over the limb of the sun, as it appears to be, the cloud is about 250,000 miles (400,000 km) above the sun's surface. There are many condensations in it easily distinguished on the Sproul and the Douglas plates. Seven

[1] See *Publications of the Astronomical Society of the Pacific*, **32**, 5, 1920.

points were identified on the two plates and measured. The point 1 in Table III was chosen and identified on both plates by Professor Miller, point 2 by Miss Onderdonk, point 3 by Professor Pitman, point 4 by Professor Marriott, who on May 7 measured the position angle from the north pole of the sun and the radial distance in inches from the sun's center. Professor Marriott chose later four additional points on plates made from different negatives from those from which the first series was made, and measured them. The measures are all shown in Table III.

TABLE III

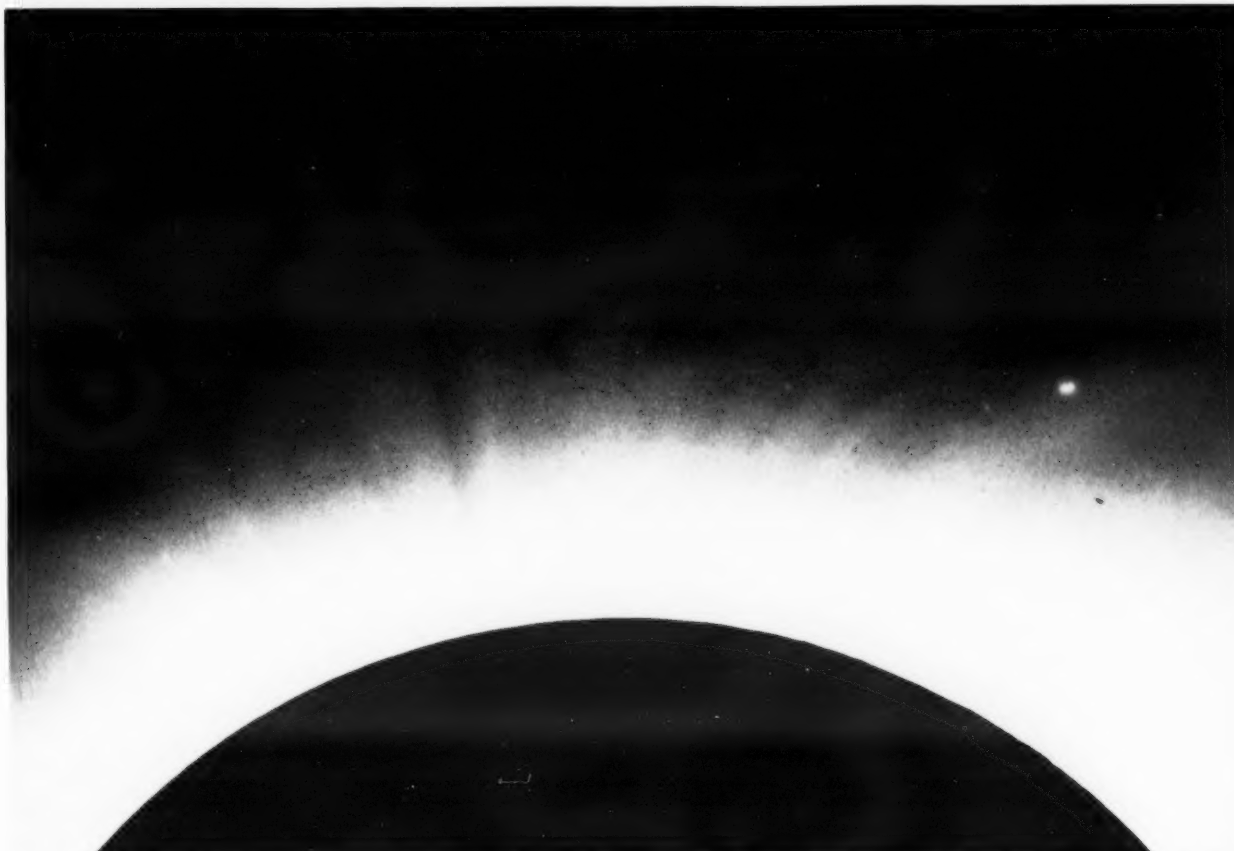
MEASURE OF CONDENSATIONS IN CLOUD

	Radial Distances				Position Angle		
	Observer	Sproul	Douglas	S−D	Sproul	Douglas	D−S
	First Series Plates						
1	Miller	4.73	4.49	0.24	118°25′	120°50′	2°25′
2	Onderdonk	4.85	4.60	0.25	119 16	121 20	2 04
3	Pitman	4.91	4.71	0.20	119 44	122 20	2 36
4	Marriott	4.74	4.52	0.22	118 40	120 56	2 16
	Second Series Plates						
1	Marriott	4.74	4.51	0.23	117 03	119 25	2 22
2	Marriott	4.75	4.48	0.27	115 15	117 50	2 35
3	Marriott	4.77	4.51	0.26	120 08	122 52	2 44
4	Marriott	5.28	5.04	0.24	117 40	120 09	2 29

These measures show that the condensations are moving outward from the sun and also drifting toward the equator. The average radial velocity (assuming the cloud is above the limb of the sun) is 19.5 miles (31.4 km) per second. The average velocity perpendicular to the radius is 19 miles (31 km) per second.

On the west limb of the sun at heliocentric latitude $-56°$, there is a large eruptive prominence whose base subtends an angle of 6° at the center of the sun. Above this prominence, a strip of the corona, on the negative, is decidedly lighter than the surrounding corona. This strip is somewhat wider at some distance above the sun than at the top of the prominence, then becomes narrower, and finally about 7′ above the solar surface, it disappears.

PLATE VI



North Polar Streamers

This phenomenon suggests that the prominence has a screening effect on that which produces the light of the corona. If this light is produced by the presence of solid or liquid particles that reflect sunlight, then these particles seem to be less frequent there. An alternative suggestion is that it is possible that the prominence screens the light of the sun from the region above it. This is not an unusual phenomenon. In fact, there is a light region on many eclipse negatives above eruptive prominences, but this one is much larger than usual. This screened region extends above the surface of the sun 125,000 miles (200,000 km), but it is not radially above it. It conforms to the shape of the streamers that border it on either side, all of which bend toward the sun's equator. In this screened area, there is a series of three slightly wavering streamers, well defined and as black (on the negative) as the corona outside the screened region.

5. *Unusual polar phenomena.*—It is impossible to see the peculiar phenomenon in the vicinity of the polar rays referred to in this paper (p. 79) without speculation. It has been shown[1] that practically any streamer hitherto photographed in the corona can be produced mechanically, provided one grants the assumptions that the streamers are composed of particles of matter ejected radially from the solar surface with a low velocity, and subject after ejection to the attractive force of the sun, and a radiation pressure which varies inversely as the square of the distance of a particle from the sun's center. With these assumptions it is easy to determine the elements of the orbits of these particles, and to compute the shape of any streamer hitherto observed. These computed streamers faithfully duplicate the observed ones until they reach a distance of two and one-half or three radii of the sun. The computed streamers then turn back abruptly. We have never found streamers on large-scale plates that extended to so great a distance, and hence have never seen streamers that turn back so abruptly. As was said above, the sheet around the north pole breaks up into many streamers, and all these streamers turn back abruptly at their extremities. Hence we believe that one interpretation is that they are the extremities of the usual coronal streamers which originate at rather low latitudes.

[1] *Astrophysical Journal*, **30**, 303, 1909.

NORTH POLAR STREAMERS

From the point of maximum curvature of the streamers a perpendicular to the axis of the sun was measured for a half-dozen streamers. To define other points on the streamers, a series of concentric arcs with a center coincident with that of the sun was drawn on the Sproul plates, and from the points where the arcs intersected the streamers, the perpendiculars to the axis of the sun were measured. The corresponding points on the Douglas plates were then determined by means of the scale relation of the Sproul and Douglas plates, and the corresponding distances on the Douglas plates measured. The largest difference, Douglas *minus* Sproul, of these measured distances was 0.03 inch (0.76 mm). Of the twenty-two determinations made, one was negative, one zero, and twenty were positive. The mean of the whole twenty-two measures was 0.015 inch (0.4 mm), and if these streamers were back of the sun indicates a rotation of the corona in the same direction and of the same order of magnitude as that of the sun.

6. *The Einstein cameras.*—To search for the Einstein effect, we had built by the James B. McDowell Co., two quadruplet lenses following the Brashear-Hastings formula. The aperture of these lenses is $6\frac{3}{4}$ inches (17.1 cm) and their focal length is approximately 15 feet (4.6 m). They are most excellent lenses. The stellar images, on plates 18 inches square, are almost as good at the corners as at the center of the plate. Exposures at Swarthmore, and also at Yerbanis, showed that on a clear night one could record with this camera, in 50 seconds, weak images of stars of 10.2 photographic magnitude. There were fifty stars in the eclipse field as bright or brighter than magnitude 9.5, and they were fairly symmetrically situated with regard to the sun. These lenses were mounted in a twin tube of structural iron, which was carried by a polar axis, also of structural iron. The motion of the cameras was controlled by a sector arm. We exposed two plates in each of these cameras (see Fig. 1 *b*). The first one was exposed on the eclipse field for 55 seconds. The telescope then was shifted 10° in declination, and an exposure of 20 seconds made on the same plate on a reference field. The plate-holders were changed and the reference field photographed on the second plate, the telescope was then shifted to the eclipse field and an exposure of 55 seconds made. The program differed from that of Campbell in taking the reference field during the eclipse. The

cameras were very carefully adjusted. The axes of the lenses met the plates at their geometrical center, and the plane of the plate was perpendicular to the axis of the lens. The plates used were coated with a Seed 30 emulsion on selected plate-glass $\frac{1}{4}$ inch thick. A plate-holder containing a plate weighed about 22 pounds. These were handled during the eclipse most successfully by Earl L. Williams and Adrian C. Rubel. The instrument was in charge of Professor Miller.

The program was carried out as planned but the results were disappointing. At the beginning of the eclipse and also at the end of the eclipse, light clouds were floating over the eclipse region, so that all the plates were affected by the clouds. We found only fourteen star images of the eclipse field, and most of these were on one side of the sun. The images were not good. We lost the entire reference field because of clouds covering that region of the sky.

The question has been raised whether an exposure of 20 seconds on a plate already exposed 55 seconds on an eclipse field will not blacken the plate so as to destroy the definition of the images. This is a real question which we cannot answer with assurance, but it is our opinion that the negative answer is the right one. We intended to photograph on each of these plates an evening field. The weather prevented this for three days before the eclipse and three days after it.

7. *Photographic determination of the moon's diameter.*—Certain deflections in the positions of stars photographed in a field immediately surrounding the sun during a total solar eclipse were found by the English observers in 1919, by Campbell and Trumpler, by Chant, and by Dodwell and Davidson in 1922. This deflection was in direction and amount nearly equal to that required by the Einstein Theory. That found by Campbell and Trumpler agreed almost exactly with that predicted by Einstein. Among many suggestions to account for these deflections is one made by Professor Charles Lane Poor in November 1919, and many times since. The suggestion is that due to the fall of temperature of the air in the shadow cone there might be, after the ordinary corrections for refraction have been made, a residue of refraction sufficient to account either partially or completely for the so-called Einstein effect. In other words, it is suggested that the bending of the rays of light occurred

not at the sun but in our own atmosphere. If this were so, it is evident that the apparent diameter of the moon at totality would be larger than the theoretical diameter. Dr. Poor suggested that the truth of this contention might be tested by measuring the diameters of the images of the moon obtained with very short exposures during the eclipse, and comparing these measured diameters with the computed theoretical diameter. He proposed that if we would make the necessary exposures, he would provide instruments suitable for making the test. Although we recognized the difficulty of measuring accurately these diameters, we believed the error of measurement would be much less than the deflection of star positions above referred to, and that the method, therefore, would furnish at least a qualitative test of the correctness of the suggestion. Since the only camera in our possession suitable for the purpose had already been assigned to the part of the program described above, an additional camera, almost an exact duplicate of the Einstein cameras, was ordered from the James B. McDowell Co. The expense of providing the additional apparatus and observers was borne by Mr. H. G. S. Noble, Mr. E. Vaile Stebbins, and Professor Poor. The lens was mounted in a structural iron tube lined with black cloth, and covered with white rubberized cloth. The tube was carried on a structural iron axis. Dr. Curtis and Dr. Alter were in charge of the camera.

Dr. Poor and ourselves agreed upon the following program: Dr. Alter exposed on a star field the night before the eclipse. On the same plate we exposed 63 seconds on the eclipse field. We then shifted the plate in the camera, and took two very short, almost instantaneous, exposures of the moon. The *plate* was shifted in order that those short exposures might be on the optical axis of the lens. The process was repeated on a second plate, but the short exposure was not a good one. The camera was carefully adjusted as to focus, as to location of the point where the optical axis cut the plate, and as to the perpendicularity of the plane of the plate to the axis of the lens. The plates were in metal plate-holders carefully made for us by the Chester Dairy Supply Co. The plates were coated with Seed 30 emulsion on selected plate-glass secured from the Pittsburgh Plate Glass Co. Had all gone well this program would have yielded

us on each of two plates a reference field, the eclipse field, and two short exposures of the moon. From these we hoped to obtain not only the diameters of the moon, but also, by using the methods applied to the Wallal plates by Campbell and Trumpler, to find such deflections of the rays of light as existed. But the weather was against us. It turned out that clouds prevented us from getting a night field either before or after the eclipse. The eclipse field, because of thin clouds at the beginning and the ending of totality, yielded us Venus and four stars, viz.: σ Leonis, 80 Leonis, 82 Leonis, and B.D. $+7^\circ 2440$. The short exposures of the moon are very good indeed. It is possible that exposures half as long would have been better. There is a suggestion of corona around the circular image, and the spread of light on the plate may have decreased the diameters slightly.

We had no measuring engine large enough to receive these plates, which are 18×18 inches (45.7 cm). Professor Schlesinger generously put at our disposal one of his measuring engines on which Professor Miller made two sets of measures of the moon's diameters. The first set was of four diameters on each of two images. Some weeks later, he measured six diameters on each of two images. The movable wire was set tangent to what seemed to the measurer to be the limb of the moon. The eyepiece of the measuring engine possesses a reversing prism. On limb I, two settings were made, one with the prism so turned that it gave a direct reading, the other a reversed reading. Then two like settings were made on limb II. Two additional settings were then made on each limb in the same order, so that four settings were made on each limb. All these settings were very accordant. All the diameters on image I were measured before any diameter was measured on image II. The diameters of the two images were not measured in the same order. Professor Miller expressed the opinion before the measures were reduced that the second set was very much better than the first set. For that reason, and because there was a greater number of diameters measured the second time, we have in the computations given the first set a weight of 1, and the second set a weight of 3. The measures are given below. The position angle means only that for a reading of $360^\circ.2$ the vertical wire of the machine was parallel to

the hour circle of the center of the moon. The position angles 330° and 240° approximate the vertical and horizontal diameters respectively.

With the plate set up so that the X-axis was parallel to the equator, we measured the X- and Y-co-ordinates of Venus and the four stars mentioned above. Also the linear distance between σ Leonis and 80 Leonis, between σ Leonis and 82 Leonis, between B.D. $+7°2440$ and 80 Leonis, between B.D. $+7°2440$ and 82 Leonis was measured directly. The linear distances between the other pairs were not measured because the distance between each pair was small and because the line joining them was nearly parallel to either the X-axis or to the Y-axis. The distances between Venus and the stars were not measured because Venus was elongated nearly parallel to the X-axis. The direct distances thus measured agreed almost exactly with those obtained by computing these distances from the measured X- and Y-co-ordinates. With these measures, the only data at our command, Professors Comrie and Marriott made a determination of the scale of the plate. The usual corrections for the catalogue position of the stars, refraction, aberration, parallax, etc., were made.

TABLE IV

First Set			Second Set		
Position Angle	Diameter Image I	Diameter Image II	Position Angle	Diameter Image I	Diameter Image II
359°	44.031 mm	44.027 mm	360°.2	44.064 mm	44.049 mm
44	44.008	44.006	330.2	44.120	44.102
89	44.133	44.124	300.2	44.046	44.093
134	44.071	44.066	270.2	44.092	44.136
			240.2	44.119	44.126
Mean	44.061	44.055	210.2	44.118	44.102
			Average	44.094	44.101

When the X-measures of the five stars were reduced, it was found that unusually large residuals were given by the plate constants deduced, and that the scale found was entirely out of agreement with all the other determinations. By a thorough process of sifting, it was found that the discordance was due to Venus and 82 Leonis, and it seemed wise to reject these two in computing the

scale values from the X-measures. The image of Venus was a very long ellipse (due to the movement of Venus during the exposure), the major axis of which was nearly in the X-direction, so that it was exceedingly difficult to determine the center of the image. The Y-measures, along the direction of the minor axis, are not subject to the same uncertainty.

We set up the camera after return to Swarthmore and made a photograph of the cluster Praesepe, and determined a scale from that plate. The result, shown in Table V, was not used in computing the angular diameter of the moon. The scale determinations, with the weights assigned after careful consideration, are:

TABLE V

Determined From	Scale Value	Weight
1. *Direct measures of distance:*		
a) σ Leonis to 80 Leonis	$45''.040 = 1$ mm	2
b) σ Leonis to 82 Leonis	45.037	2
c) 80 Leonis to B.D. 7°2440	45.045	2
d) 82 Leonis to B.D. 7°2440	45.039	2
2. *"Y"-measures of five stars*	45.034	3
3. *"X"-measures of σ Leonis, 80 Leonis, and B.D. 7°2440:*		
a) Forward measures	45.039	1
b) Backward measures	45.045	1
Weighted mean	$45.039 \pm .001$	....
Scale from Praesepe plate	45.030	....

Combining this scale with the mean measured diameters to which the differential refraction was applied, we obtained values for the diameter of the moon, as follows:

The first set of measures . . . . $1984''.93$, weight 1
The second set of measures . . . . 1986.67, weight 3

Hence the weighted mean diameter of the moon is $1986''.2 \pm 0''.30$
and the weighted mean semi-diameter is $993''.1 \pm 0''.15$

This semi-diameter may be compared with either of two computed semi-diameters, namely, either the eclipse semi-diameter, which is the distance from the center of the moon to the bottom of the valley on the moon's limb, or the occultation semi-diameter, which is a mean diameter. It was clear that our mean should be compared with the occultation diameter. Director W. S. Eichel-

berger at the request of Dr. Poor computed the occultation semi-diameter to be 992".96. The difference between the observed and this computed semi-diameter is 0".14. The computations were made by Professors Comrie and Marriott.

There is no doubt that the method proposed by Dr. Poor furnishes a practicable test of his theory. The measures indicate a slight radial expansion of the moon, though the quantity found is about equal to the probable error. If the second set of diameters only are considered, the difference, observed diameter *minus* the computed diameter, is 0".37. It is our belief that this is nearer the truth than the quantity given above. These results can be considered as preliminary only. As indicated previously, we were unable to obtain a night field, and the thin clouds which partially obscured the eclipse region during a part of each of the exposures blotted out the images of the fainter stars of the eclipse field. The plate constants and the scale of the plate were therefore obtained from measures of four stars and of Venus, all of which were on the same side of the center. The accordance of results for the plate constants leads us to believe that there exists no large error in the scale of the plate. The short exposures on the moon were a little too long. There is a fringe of corona and some spreading of light. There is no doubt that the diameters were not measured too large, and it is probable that they were measured too small. We intend to repeat this experiment on January 24, 1925, making the exposure of the moon practically instantaneous.

At 2:00 P.M. on September 10 the temperature was 74° F., the barometer read 23.46 inches, and the relative humidity was 70.

8. *The spectroscopic program.*—The spectroscopic program was limited to two problems. We attempted with objective gratings to obtain the flash spectrum and the green coronal ring. Dr. Curtis suggested that an attempt be made to obtain a spectrum of the flash in the infra-red. He designed and built an instrument for this purpose. He utilized a Rowland concave grating in the possession of the Allegheny Observatory, which was originally ruled for Professor Langley and used by him in his earlier bolometric work. This beautiful grating has about 3600 lines to the inch, with a focal length of only 34 inches (86 cm), and the ruled surface is 5.17×3.15 inches (13.2×8 cm). It gives brilliant spectra, and,

from its brilliancy, short focal length, and relatively coarse ruling, it appears to be particularly adapted to the difficult problem offered in the study of the flash spectrum in the longer wave-lengths.

The spectrograph is built of aluminum plates 0.1 inch in thickness, forming a box 9 inches deep, 39 inches long, 10 inches wide at the grating end, and widening to 18 inches at the end containing the aperture and the plate-holder; to cut out stray light a rectangular tube extension of lighter aluminum plate is attached to this end, 22 inches long and 8×5 inches in cross-section; both the box proper and the extension are fitted with appropriate diaphragms, and an aluminum shutter worked by a cord is provided at the outer end of the extension.

The instrument was designed to work in the first order from about λ 5700 on, and as the angle of deviation is only $6° 46'$, some special features were necessary in the design of the plate-holder mechanism so as not to cut off any light from the incident beam; this portion of the spectrograph is made of brass. A longitudinal slide and scale provide for focal adjustment; in addition there is a rotation about a vertical axis in the plane of the film so as to permit different tilts. The slide carrying the plate-holder has a movement of nearly 4 inches vertically by a rack and pinion motion so designed that one revolution of the pinion gives a movement of slightly less than 1 inch, thus providing for four exposures by shifting the plate-holder. The small crank which actuates the pinion protrudes through the side of the box; it is provided with a "click" stop to make a single rotation easy and certain, and an additional ratchet stop makes it impossible for an excited or forgetful observer to turn the handle the wrong way.

Eastman Commercial Ortho films, $3\frac{1}{4} \times 4\frac{1}{2}$ inches in size and stained with dicyanin, were used in a metal plate-holder. The films were held to the proper curvature against a concave metal plate by a convex brass grid with four rectangular openings $3 \times \frac{13}{16}$ inches in size. To the front of the framework of the plate-holder, a metal frame was attached carrying a celluloid "A" filter to cut out the violet of the second order.

A strong framework fastened to the polar axis oriented the instrument so that the tangent to the cusps should be parallel to the lines of the grating. The instrument was worked during the eclipse by

Professor Long, of Franklin and Marshall College. The program provided for an exposure on the first flash, a 20-second and a 150-second exposure on the corona, ending with an exposure on the second flash at third contact.

The film was badly blackened by the faint remnants of turpentine vapor from the paint on the inside of the drying box, which, though constructed a week before and liberally aired, had evidently not completely dried out in the stormy weather which preceded the eclipse. Because of this it was necessary to reduce the film by the ferricyanide method. There is no record of the first flash, nor are there any certain traces of coronal rings on the exposures made during totality. The second flash is recorded, but, because of a slight delay in starting the last exposure, ran a second or two too long after totality ended, so that the continuous spectrum masks the fainter and shorter arcs in the red region of the flash. This continuous spectrum extends to λ 8500, though very faint beyond λ 8000. Hα shows very strongly in an arc of about 56°. There is nothing comparable with it in this region, and it seems certain that there is no very strong radiation between Hα and λ 8500 in the solar flash spectrum. The helium lines at λ 7065 and λ 7281 can be made out, extending a little beyond the continuous spectrum, in arcs of about 21°. A number of other lines are suspected but cannot be made out with certainty.

The instrument seems well adapted for investigation of this region, and it is hoped that conditions may be favorable for its use in 1925.

A second objective grating was in charge of Professor Wright. This grating has a focal length of 7.5 feet (2.3 m). It is ruled 15,000 lines to the inch and so adjusted that it recorded the spectrum on the blue side of λ 5000. The grating and the plate-holder were mounted in a wooden box, which was carried on a polar axis. The sun was received directly through an opening in the side of the box. The plate-holder contained three films so arranged that we could make two exposures on the first flash, and one on the corona during totality. The grating was not in sharp focus on the day of the eclipse, due, we believe, to the fact that the adjustment, which was made on the stars several days before, had been disturbed by the

excessive rains of the three days preceding the eclipse. While the spectrum of the flash contains several lines, they are not of good quality.

An attempt was made by Professor Wright to obtain the rotational velocity of the corona by the method used by Buisson, Fabry, and Bourget,[1] in their study of the Orion nebula. The etalon had a plate separation of 2.5 mm, and the lens produced a magnification of nearly 3.5. The etalon had a free aperture of 1.5 cm, and the camera a focal length of 25 cm. It was hoped to isolate the green coronium ring system by the use of Kodak filter 12, which is opaque below about 5100 A, and the special green sensitive plate made by the Research Laboratory of the Eastman Kodak Co. Portions of the ring system of the neon yellow line were registered on the same plate as that exposed to the corona to serve for comparison. Two exposures were made, one of 120 seconds, and one of 30 seconds. The coronal image on each was very much overexposed, and no sign of rings was found. Subsequent reduction of the image with ferricyanide also failed to reveal them.

We desired, if possible, to get some definite information regarding the shadow bands. It also seemed desirable to have a motion picture made of the eclipse from the first contact to the last, as well as some of the operations of the instruments during the time of the eclipse. We presented this matter to the Pathé News who, in a way characteristic of the enterprise of this organization, undertook the work. They secured an extremely rapid motion-picture lens for photographing the shadow bands. We spread a white sheet upon the ground on which was marked a scale in feet, and a north and south line. The face of a watch was so focused by a lens that in one corner of the photograph was recorded the time of exposure, and on the remainder of the film was recorded the scale, the north and south line, and, we hoped, the shadow bands. In this way we hoped to get the distance between two shadow bands, the direction, and the velocity with which they moved. A great many feet of film were exposed, but the shadow bands at this eclipse were extremely faint, perhaps due to clouds, and the film did not show any trace of them. Visual observations of the shadow bands were made by various members of the observing staff. Mr. Charles Charlton,

[1] *Astrophysical Journal*, **40**, 241, 1914.

of the Pathé News, in co-operation with the observers of the expedition, made a motion picture of the erection, adjustment, and operation of the instruments; also of various activities of the camp, and on the day of the eclipse a motion picture of the entire eclipse, from first to last contact. The picture gives a very realistic impression of the phenomenon. It has been shown in motion-picture theaters throughout the entire country. This picture is the property of the Pathé News, but they furnished us copies of the entire film made there, and we are at liberty to permit its use for other than commercial purposes.

During the entire seven weeks, we lived in tents which we had taken with us. They were in close proximity to the camp, which was built on a long sweeping slope over which some of the battles of the Mexican revolution had been fought. This slope was covered with cacti and thorny shrubs. We lived undisturbed by the telephone and other modern luxuries; even the radio worked badly. Three days elapsed after the death of President Harding before we knew of it; however, we gathered in the dining-hall, at the hour appointed, to participate with our fellow-countrymen in the memorial services to our departed president. We had many visitors; the Mexican party under the leadership of that prince of good fellows, Director Gallo, was next door. We were honored on two occasions by having as guests directors of the German eclipse party, and many times we were visited by cultured Mexicans, and also by Americans living or traveling in Mexico. We had this advantage over our California colleagues—we were known as *the* American Scientific Expedition. The Mexican people were very courteous and helped us in many ways. We left our camp unguarded and the inconveniences we suffered were trivial. We did not succeed in every problem we attempted, but we did succeed in getting many excellent photographs of the corona with varying focal lengths and exposures. We partially succeeded in other parts of the program, and gained excellent experience in many things. We shall forget our privations, but shall remember always with keenest pleasure the fine fellowship shown by every member of the party.

SPROUL OBSERVATORY
SWARTHMORE COLLEGE
July 9, 1924

GRAVITATIONAL FORCES IN SPIRAL NEBULAE

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ABSTRACT

Numerical analysis.—An analysis is made of the radial and rotational components and of the angular internal velocities of the spirals M 33, 81, 51, 63, 94, 101, N.G.C. 2403, measured by van Maanen. The conclusion is drawn that only along the arms are the velocities tangential to those arms, that the angular velocities in each spiral have the same sign and diminish from the nucleus outward, and that the velocities along the radius may have either sign and be of any magnitude outside of the arms.

Hypotheses.—It is shown that the average velocities, angular and along the radius, at different distances can be explained on the assumption that the nebula consists of a nucleus of concentrated matter together with a widely extended distribution of low space density, in which collisions are rare. The particles or masses constituting this distribution are supposed to be describing orbits under the attraction of the matter present. To account for the arms, the further hypothesis is made that these orbits are of such a character that their envelopes are the arms of the spiral and that the latter structure shows on account of the resulting concentration of matter on the envelope.

Conclusions.—The space densities of the distributed matter are of the order 10^{-15} that of the sun, and their masses, with a common parallax of $0\overset{''}{.}001$, of the order 10^8; the masses of the nuclei are smaller but of the same order of magnitude.

1. The paper by Mr. J. H. Jeans[1] on "The Internal Motions of the Spiral Nebulae" brings forward the difficulties which are encountered when an attempt is made to discover the forces under which these motions are produced. Assuming that the motions in general are along equiangular spirals outward, he computes the forces which are necessary in order to produce them and shows that not only are they non-central but that the laws of their variation cannot arise from any known source. This paper is an attempt to explain the internal motions by gravitational action alone, with the aid of certain assumptions concerning the distribution of the matter and the orbits described by its various portions.

VELOCITIES ALONG THE RADIUS

2. The crucial point in Jeans's investigation is his conclusion that the motions of all points of the nebula are, in general, along equiangular spirals, and before we can test any hypothesis this conclusion must be further considered. For this purpose Table I has been constructed, containing for each measured nebula (except M 63) the number of positive and negative values of the radial

[1] *Monthly Notices, R.A.S.*, **84**, 60, 1923.

components given by van Maanen with their respective means. The methods used to obtain these from van Maanen's papers[1] are explained in section 5 below.

In three of these objects the ratio of the number of points with positive velocities along the radius to that with negative values is about 2 to 1, in one it is 3 to 1, and in two others it is much larger. In my view of the measures these differences are largely accounted for by their distribution, the frequency being in general much greater along the arms than elsewhere and that, had the distribution been more uniform, the inequalities in numbers would have largely disappeared.

TABLE I

RADIAL COMPONENTS

Nebula	No. Pos.	Mean	No. Neg.	Mean
M 33	229	$+0\overset{''}{.}008$	129	$-0\overset{''}{.}007$
M 81	59	.022	5	.014
M 51	58	.013	19	.008
M 63	....	....	....	....
N.G.C. 2403	68	.016	5	.010
M 94	22	.022	10	.011
M 101	59	+0.016	28	−0.011

3. That the means of the negative velocities are numerically somewhat smaller than those of the positive velocities is, with the hypothesis of this paper, also a consequence of the distribution. According to the theory developed in section 17 following, the radial velocities along the arms are always outward and tend to have larger mean values there than elsewhere, so that the negative velocities would show general mean values more accurately than the former. This view is strengthened by the case of M 33, for which the measures are much more numerous and better distributed than in the cases of the other nebulae. Table II, which is an extension of Table I for the case of M 33, gives the number of positive and negative velocities along the radius at different distances with their means, and illustrates these conclusions.

It is noticeable that the chief differences in the numbers of positive and negative radial components lie in the region between

[1] *Astrophysical Journal*, **57**, 64; **54**, 237, 347; **56**, 200, 208; **57**, 49; **44**, 218 (*Mt. Wilson Contributions*, Nos. 260, 213, 214, 242, 243, 255, 118).

$r=2'$ and $r=8'$, and it is in this region where the arms are most clearly marked and where, in the diagram of velocities given by van Maanen, the measures along the arms appear to be relatively most numerous. Indeed, outside of this region, the number of negative velocities is greater than that of the positive velocities. It is further evident that in general the mean values of the positive velocities are somewhat greater than those of the negative velocities within this same region only, the result being reversed outside. The reason for this reversal is not evident: one would expect the means to be approximately the same in the outer region.

TABLE II

M 33, RADIAL COMPONENTS

r	No. Pos.	Mean	No. Neg.	Mean	r	No. Pos.	Mean	No. Neg.	Mean
1′5.....	8	+0″009	4	−0″006	8′5.....	15	+0″009	11	−0″008
2.5.....	12	.007	1	.005	9.5.....	20	.009	15	.006
3.5.....	26	.008	5	.003	10.5.....	13	.004	18	.007
4.5.....	26	.009	3	.011	11.5.....	10	.005	21	.007
5.5.....	29	.008	10	.007	12.5.....	11	.007	17	.009
6.5.....	25	.009	4	.006	13.5.....	1	.006	7	.011
7.5.....	33	+0.008	13	−0.006					
					Total..	229	+0.008	129	−0.007

I therefore conclude that while the points on the arms have outward motions tangential to those arms, the motions in other parts of the spiral may be either inward or outward.

4. The rotational velocities are, in general, all of the same sign. The numbers of points with signs opposite to those of the general rotation are in the seven cases, respectively, 3, 1, 5, 5, 11, 3, 9. These may certainly be regarded as accidental errors of measurement. Some doubt might arise in the cases of N.G.C. 2403 and M 101, but in both these cases van Maanen has measured two pairs of plates and the discordances are sufficient evidence of their accidental character.

CLASSIFICATION OF THE VELOCITIES

5. While the evidence furnished by the averages all goes to show their remarkable accuracy from the point of view of measurement, the errors, owing to the minute quantities involved, are large compared with the actual velocities. It therefore seems possible

only to make deductions by means of averages. The particular form which these averages shall take depends on the theory to which reference is made. For the purposes of this paper, I need only the radial and rotational components, and the average values of these have been given by van Maanen for various values of the radial distance. But for testing the gravitational hypothesis we need the mean values of the velocities along the radius without regard to sign, and these have been obtained directly from van Maanen's detailed measures.

In the cases of M 33, M 51, and M 81, a classification more detailed than that given by him seemed advisable to bring out more clearly the variations of velocity with distance from the nucleus and also to exhibit the general magnitudes of the accidental errors on the assumption that these variations should follow a uniform law. For M 81, which is considerably inclined to the plane of projection, each point was plotted on a diagram and ellipses were drawn according to the position of its projected plane as estimated by van Maanen, so that the distances might be read off; he has given the velocities after making this correction. This procedure was probably unnecessary since the use made of the data here involves the averaging of deviations of the same nature as those due to projection. Thus, for M 51, which is also inclined to the plane of projection at a sensible angle, the distances were obtained directly from the projected co-ordinates as given by van Maanen. The mean velocities along the radius in the case of M 81 were obtained only to the nearest multiple of $0.''005$.

For M 63 the velocities along and transverse to the stream are alone available. Instead of deducing those radial and transversal, it was considered sufficient to use the stream velocities as rotational, since the motion appears to be nearly circular.

An additional classification of M 33 according to each $0.''001$ of the radial component from $+0.''020$ to $-0.''020$ was made, showing the distance and rotational velocity for each value, and the forty-one lists were averaged. But no result appeared which could not better be deduced from the classification according to values of the distance, and the results are therefore omitted. A possible exception is the lists of values for v_r (the radial component) $=0.''000$, $\pm 0.''001$, which are mentioned in section 22.

The means for N.G.C. 2403, M 94, and M 101 are those of van Maanen except for the change to the absolute values of the velocities along the radius.

6. Table III gives the averages obtained as explained above for each of the seven nebulae. The first column contains the mean distance r for the numbers on the same line in minutes of arc; the second and third, the mean rotational $(r\omega)$ and radial (v_r) components, the latter all taken positively, in units of annual motions of 0.″001; the fourth, the number of points used; the fifth, the angular velocity (ω) deduced from the first two columns and therefore expressed as radians per 60,000 years; the last column contains the corresponding value of ω as calculated from the formula attached to each heading. The numbers in brackets are computed from a formula given in section 11.

MOTION UNDER CENTRAL FORCES

7. If a particle moves in one plane under a central force, its mean angular velocity depends mainly on the law of variation of the force with the distance while the ratio of the radial to the rotational velocity depends principally on the shape of the orbit. If, therefore, we have numerous particles describing orbits in the same sense, and the velocities of those particles are alone given, the unknown law of force will be determined mainly from the angular velocities. In attempting to construct a distribution of matter which will account for the observed velocities, angular and along the radius, in a spiral nebula, I shall leave aside at the outset the apparent inequalities of distribution of the matter as we go round the axis of the spiral. This amounts to regarding the characteristic visible feature of the spiral as a secondary effect, the primary consideration being the distribution of the matter with reference to the distance from the axis.

The simplified problem is this: Assuming that the nebula consists of particles or masses of matter in rotation about an axis and that at any instant the matter is uniformly distributed round any circle with its center on the axis, to find what distribution will give the observed angular velocities. Since numerous observations of various spirals leave little doubt that the internal motions in any one must be nearly all in the same plane, it will be possible

TABLE III

r	$r\omega$	v_r	n	ω (Obs.)	ω (Cal.)
M 33 $\omega^2=3.5+800\,r^{-3}$ to $r=13.5$					
1.5	12	8	12	8	17
2.5	11	7	13	5	7
3.5	13	7	31	4	5
4.5	16	8	30	3.5	3.6
5.5	17	8	40	3.1	3.0
6.5	20	9	25	3.2	2.6
7.5	21	7	49	2.8	2.4
8.5	21	8	28	2.5	2.3
9.5	19	8	38	2.0	2.2
10.5	22	6	33	2.1	2.2
11.5	20	6	33	1.7	2.1
12.5	25	8	28	2.0	2.0
13.5	23	9	10	1.8	1.9
M 51 $\omega^2=90+100\,r^{-3}$ to $r=2.75$					
0.25	7	22	1	28	80
0.75	11	13	3	15	15
1.25	20	9	11	16	12
1.75	15	7	7	9	10
2.25	22	13	24	10	10
2.75	21	12	17	8	(10)
3.25	24	12	4	7	(6)
3.75	17	13	3	5	(5)
4.25	17	14	3	4	(4)
5.5	38	14	1	7	(3)
M 81 $\omega^2=24+2000\,r^{-3}$ to $r=10.5$					
3.5	21	20	2	6	8
4.5	36	20	8	8	7
5.5	34	20	26	6	6
6.5	35	20	29	5	6
7.5	36	15	23	5	5
8.5	46	20	8	5	5
9.5	51	10	4	5	5
10.5	55	40	3	5	5
N.G.C. 2403 $\omega^2=21+120\,r^{-3}$ to $r=4.0$					
1.4	11	16	23	8	8
3.1	15	12	37	5	5
5.1	15	19	11	3	(3)
7.3	23	23	3	3	(2)
8.9	28	30	2	3	(2)

TABLE III—*Continued*

r	$r\omega$	v_r	n	ω (Obs.)	ω (Cal.)
M 94 $\omega^2=96+200\,r^{-3}$ to $r=2.0$					
0.7	18	15	19	26	26
2.0	22	20	13	11	11
M 63 $\omega^2=21+420\,r^{-3}$ to $r=4.5$					
0.5	16	....	11	32	58
1.5	18	....	36	12	12
2.5	20	....	32	8	7
3.5	22	....	12	6	6
4.5	24	....	3	5	5
M 101 $\omega^2=3+2000\,r^{-3}$ to $r=5.0$					
2.0	24	10	15	12	16
4.0	27	11	32	7	6
6.0	16	20	16	2.7	(2.3)
9.0	16	16	24	1.8	(1.9)

further to simplify the problem by assuming that there is also symmetry with respect to the equatorial plane of the nebula and that the motion of any particle is substantially the same as that of a particle in the equatorial plane.

8. While the mean observed angular velocities diminish in general as we go outward from the nucleus, they certainly do not diminish according to the three-halves inverse power of the distance, which a predominant mass at the nucleus would require. In fact, the tendency is to diminish rapidly at first, and then to become nearly constant at some distance from the nucleus. A constant mean angular velocity at all distances is characteristic of a central force which varies directly as the distance from the center, and such a force is produced in the equatorial plane of an ellipsoid of uniformly distributed matter on a particle within the mass.

Let us assume, therefore, that the matter in the nebula consists of a concentrated nucleus surrounded by a uniform distribution of matter in the form of a flattened ellipsoid of revolution with its center at the nucleus, the masses and dimensions of the system to be determined, if possible, from the observations. It is further

assumed that every mass or particle of the ellipsoidal distribution is in motion in the same sense round the nucleus but that collisions are rare, so that each particle may be assumed to be moving freely under the attraction of the whole system whether it be inside or outside the ellipsoid.

9. Let M be the mass of the nucleus and μ a constant depending on the ratio of the polar and equatorial axes and on the density within the ellipsoid, supposed to be constant. The force in the equatorial plane within the ellipsoid is then central and equal to $\mu r+Mr^{-2}$ at a distance r, the proper units being chosen.

The equations of motion of a particle describing an orbit under this law of force are

$$\frac{d^2r}{dt^2}-r\left(\frac{d\theta}{dt}\right)^2=-\mu r-\frac{M}{r^2}\ ,\quad \frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)=0\ ,$$

the first integrals being

$$r^2\frac{d\theta}{dt}=h\ ,\quad \left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\theta}{dt}\right)^2=\frac{2M}{r}-\mu r^2-C\ ,$$

where h, C, are arbitrary constants.

Our present knowledge of the orbit of any particle is confined to a knowledge of two positions very near together, that is, to its velocities in a single position. This information will give only the arbitrary constants of its orbit and tells nothing about the force under which the orbit is described. But if we assume an approximately uniform distribution of particles in an orbit and a uniform distribution of orbits round the nucleus, some further information is available.

The differential equation to the orbit, namely,

$$\left(\frac{du}{d\theta}\right)^2+u^2=\frac{1}{h^2}\left(2Mu-\frac{\mu}{u^2}-C\right)\ ,\quad u=\frac{1}{r}\ ,$$

shows that a third constant $\tilde{\omega}$ is present only in the form $\theta-\tilde{\omega}$. The remaining constant is the epoch and it does not concern us here. Hence if a be the constant of distance in any orbit and e a constant defining its shape, the equation to the orbit may be written

$$r=a\ f(\theta-\tilde{\omega},\ e,\ a/a_0)\ ,$$

where a_0 is a length constant which is the same for all orbits. Since r is assumed to be an oscillating function and θ is supposed to make complete revolutions, f will consist of periodic terms and a constant term and we can choose the latter to be unity, so that a is the mean value of r in any orbit, taken with respect to θ. Since $dr/dt = -h\,du/d\theta$, it follows that the mean value of $v_r = dr/dt$ is zero.

Tables I and II illustrate this result. Owing to the inequalities of distribution of the measures as explained in sections 2 and 3, we are to expect the means of the positive velocities along the radius to be numerically greater than those of the negative velocities, but, in the case of M 33, where the distribution is much more nearly uniform, they are nearly the same.

10. The first equation of motion may be written

$$\omega^2 - \mu - \frac{M}{r^3} = \frac{1}{r}\frac{d^2r}{dt^2},$$

where ω has been put for $d\theta/dt$. Further,

$$\frac{1}{r}\frac{d^2r}{dt^2} = -h^2\frac{d}{d\theta}\left[u^3\frac{du}{d\theta}\right] + \tfrac{3}{4}h^2\left[\frac{d}{d\theta}u^2\right]^2.$$

In the right-hand member of this last equation, the first term will have a zero mean value. That of the second term will depend only on a/a_0, e. In a circular orbit the mean value of this latter term will vanish. In a non-circular orbit this mean value will depend on the square of the deviation from a circular orbit, the deviation being expressed as a ratio to the mean value. If the orbits have nearly the same shape, the greater part of this mean value can be supposed to be absorbed in the constant μ. As we are about to obtain means at different distances which include particles belonging to orbits of different distances and shapes, the variations in such means from a constant will still tend to be small. I conclude therefore that an approximation to ω can be obtained by computing the mean value of

$$\omega^2 = \mu + Mr^{-3},$$

for various values of r.

11. The values of μ, M, are to be found for each nebula from the observations. Owing to the rough nature of the observational material and of the theoretical investigation, the use of least squares in finding them is superfluous, and I have chosen their values in such a manner as to fit reasonably the observed means; the adopted formulae are given at the head of each group in Table III.

The limiting value in each case is the estimated equatorial radius of the ellipsoidal distribution. Except in the case of M 51, there is little on which to base it. That given may, in general, be regarded as the smallest value which appeared to be possible.

Outside the ellipsoidal distribution, the formula for the force to the center is

$$\frac{M}{r^2}-2\pi\rho r\frac{r_0^2r_1}{r_0^2-r_1^2}\left\{\frac{(r^2+r_1^2-r_0^2)^{\frac{1}{2}}}{r^2}-\frac{1}{(r_0^2-r_1^2)^{\frac{1}{2}}}\tan^{-1}\left(\frac{r_1^2-r_0^2}{r^2+r_1^2-r_0^2}\right)\right\}$$

where r_0, r_1, are the equatorial and polar radii of the distribution, respectively, and ρ is its density. It is estimated that r_1/r_0 is at least $\frac{1}{10}$, so that we may put r_0^2 for $r_0^2-r_1^2$ without sensible error. At the surface and inside this reduces approximately to

$$\frac{M}{r^2}+\pi^2\rho r\frac{r_1}{r_0},$$

so that μ, expressed in the proper units, is equal to $\pi^2\rho r_1/r_0$.

The numbers in brackets in the last column of Table III have been computed from the former of these expressions with r_1^2 neglected.

12. The comparisons of the last two columns of Table III show that the assumed value for ω is at least a fairly good representation of the observations. At the same time, it must be remembered that we have had three constants at our disposal, namely, μ, M, r_0, that is, two constants to take care of the variations of ω in each nebula. The large deviations close to the nucleus are to be expected for both observational and theoretical reasons. On the hypothesis of this paper, the best we can hope to do is to obtain the orders of magnitude of μ, M, r_0, except perhaps in the case of M 33 where

the determinations of μ, M, are rather more definite. The minimum value of r_0 is least definite in the case of M 101; the observations are nearly as well satisfied for a value $r_0 > 9$ as for the chosen value 5. This is interesting in connection with the ratio of the masses of the ellipsoidal distributions to those of the nuclei mentioned in section 16.

13. Some general remarks on the orbital motions may perhaps give a clearer view of the assumptions which lie at the basis of these comparisons. If the nucleus contained practically all of the attracting matter, the orbits would be ellipses with the nucleus in one focus, and the mean angular velocities would follow Kepler's third law. This would have to be modified in the comparisons, however, if the ellipses had high eccentricities, since we are not comparing the mean angular velocities of each particle taken over the whole of its orbit, but are taking the angular velocities of all particles at a given instant and at a given distance from the center, and forming their mean.

If the ellipsoidal distribution alone existed, the orbits within this distribution would still be ellipses but their centers would be at the nucleus and the mean angular velocity in each orbit would be the same, whatever the distance or eccentricity. Just outside the distribution the law of force is complicated but tends toward the inverse square as the distance increases. The estimates of mass deduced in section 15 seem to indicate that the mass of the distributed matter is, in general, several times that of the nucleus, so that, except close to the latter, the orbits will tend to be central rather than focal ellipses in a first approximation, and in the middle portions of the spiral the central ellipse would probably furnish the best first approximation.

14. The comparison of the mean velocities along the radius with the mean rotational velocities gives some indication of the order of magnitude of the eccentricities of these ellipses. The latter are apparently nearer unity than zero, a result to be expected if the distributed matter has been ejected from the nucleus. One would expect that this ratio would decrease markedly toward the confines of the nebula on account of the increase in the relative number of apses there. In M 33, M 81, the decrease is noticeable,

but it is doubtful or absent in the remaining nebulae. The effect of the distribution of the measures may partially account for the latter, but some further explanation is needed under the present hypothesis.

THE MASSES AND DENSITIES

15. Let r, ω, represent the actual numbers present in columns 1 and 5 or 6 of Table III, and let p be the parallax in seconds of arc. Then, in terms of the year and radius of the earth's orbit,

$$\text{angular vel.} = \omega/60{,}000 , \quad \text{radial dist.} = 60\, r/p .$$

If $2r_1$, $2r_0$, be the polar and equatorial axes of the ellipsoidal distribution reckoned in the same units as r and ρ its density, the formula for μ is (section 11) $\mu = \pi^2 \rho r_1/r_0$, which gives the density when μ, r_1/r_0 are known. If μ, M, are the actual numbers in the expressions for $\omega^2 = \mu + Mr^{-3}$ in Table III, the masses of the ellipsoidal distribution and of the nucleus and the space density of the former are respectively given by

$$\frac{8}{\pi} 10^{-5} \frac{\mu r_0^3}{p^3} , \quad \frac{6}{p^3} 10^{-5} M , \quad 1.5 \times 10^{-17} \mu .$$

To obtain estimates for the densities, we must know the ratio r_1/r_0. This is probably of the order $\frac{1}{10}$, but in Table IV I give $\rho r_1/r_0$. The deduced masses are practically independent of the assumed value of this ratio.

The parallaxes of these nebulae have been discussed on several occasions and the various published estimates appear to range between 0ʺ001, 0ʺ0001. In order to make future comparisons easy, I have computed all the masses for $p = 0''001$. The estimates of r_0 have been discussed in section 11.

16. It is evident that the relative variation in mass is similar to that which occurs in the stars. The last column, which gives the ratio of the ellipsoidal distribution to that of the nucleus, shows but little variation between the various nebulae, except for M 101. But it was pointed out in section 11 that any value for r_0 greater than 5 will satisfy the observations nearly as well; if $r_0 = 15$, the ratio becomes 3 instead of 0.1.

The order of magnitude of the total mass, which has also been discussed by others, is largely independent of the particular assumption as to its distribution. It depends mainly on the external angular velocities and on the assumed parallax.

The space densities are somewhat high. If we assume that $r_0 = 10\ r_1$, they correspond to an amount of matter equal to that of the sun in a cube whose side is about a thousand times the radius of the earth's orbit. Nevertheless, the density appears to be

TABLE IV

Neb.	μ	r_0	Ellipsoidal Mass $\times 10^{-7}$	M	Nuclear Mass $\times 10^{-7}$	$\rho_{r_0}^{r_1} 10^{16}$	Ratio of Masses
M 33	3.5	13.5	20	800	5	0.5	4
M 51	90	2.75	5	100	0.6	13	8
M 81	24	10.5	66	2000	12	4	5
N.G.C. 2403	21	4.0	3	120	0.7	3	4
M 94	96	2.0	2	200	1	14	2
M 63	21	5.0	7	420	3	3	2
M 101	3	5.0	1	2000	12	3	0.1

sufficiently low to permit of the matter being distributed in such a way that collisions would be rare even with the numerous intersecting orbits, as required by the hypothesis.

THE SPIRAL ARMS

17. The foregoing discussion takes account only of differences of structure and motion of the nebula in a direction radial from the nucleus. It is now necessary to see if a development of the hypothesis can account for differences of structure in an angular direction, that is, for the spiral arms, and for one of the principal deductions from van Maanen's work, namely, that the motion of a point on an arm is, in general, tangential to the arm.

As pointed out earlier, our knowledge of the motion of a point is confined to a knowledge of its velocity—we know nothing about the curvature of its path. Jeans concludes, on account of the existence of large numbers of spirals of similar form, that the actual paths must be spirals. It seems of interest to examine another hypothesis which will account for our knowledge of the spiral motion equally well, namely, that the motion of a point is tangential

to a spiral arm when it reaches the arm, that is, that the arms are the envelopes of the paths followed by the particles or masses. This hypothesis permits and indeed requires the existence of negative radial velocities elsewhere.

From this point of view the visible structure is due to the greater space density of visible matter in the neighborhood of the arms than elsewhere, since we have assumed that collisions are rare. As we have also assumed a distribution of matter along each orbit, the "knots" will have in general to be explained by the existence of a group of orbits near together which contain additional visible matter. The difficulties in this latter assumption can be somewhat diminished by making use of the fact that the periods in the central orbits are nearly the same; the concentration could then be in a portion of each of these orbits making the knots temporary or perhaps periodic features.

One major difficulty is avoided. The assumption of a general outward spiral motion practically requires the development of all these nebulae within a period of time of the order of some hundreds of thousands of years. Since secular changes in elliptic orbits have in general periods very long compared with the periods of revolution in the orbits, the time of development is lengthened by a factor which may be several powers of 10, separating the initial stages of the various nebulae to a corresponding degree.

The assumption in the earlier part of this paper that the force at any point is central and a function of the distance from the nucleus will require modification when the distribution assumed in order to account for the arms exists. But this modification may be regarded as a second-order effect from the point of view of perturbations. Its disturbing effect will be less in those nebulae in which the "coiling" is more fully developed.

ENVELOPES

18. Every plane orbit contains four, or apart from the constant of epoch, three arbitrary parameters which for the purposes of description may be taken to be its scale (mean distance), shape (eccentricity), and orientation with respect to some fixed line in its plane (longitude of the apse). In order to get an envelope of

a group of such orbits, two relations between these three constants must be satisfied. At the outset, therefore, the envelope hypothesis places very considerable limitations on the orbital motions. Of the two relations, one is determined if we know the equation to the envelope: in the present case, this is assumed to be approximately an equiangular spiral. There does not appear to be any way of deducing the other from the observational material. The choice made below is arbitrary and was adopted partly because of its simplicity and partly because it gives an envelope which is to some extent independent of the shape of the orbit.

19. Suppose we have a triple infinity of orbits defined by the polar equation

$$r=a\ f(\theta-\tilde{\omega}, e)\ , \tag{1}$$

the arbitrary parameters being a (scale), $\tilde{\omega}$ (orientation), e (shape). The envelope of the system is obtained by eliminating a, e, $\tilde{\omega}$, from (1) with

$$f\ da+a\frac{\delta f}{\delta e}de=a\frac{\delta f}{\delta\theta}d\tilde{\omega}\ , \tag{2}$$

and two further relations between a, e, $\tilde{\omega}$.

The envelope can be the spiral

$$r=b\ \text{exp.}\ k\theta\ ,$$

provided the assumed relations between a, e, $\tilde{\omega}$, are given by

$$e=\text{const.}\ ,\quad a=a_0\ \text{exp.}\ k\ \tilde{\omega}\ ,$$

where a_0 is a constant. This result is independent of the form of f.

Hence the envelope of a system of similar curves will be an equiangular spiral provided the scale (a) and the orientation ($\tilde{\omega}$) are connected by the relation

$$a=a_0\ \text{exp.}\ k\ \tilde{\omega}\ ,$$

where a_0, k, are common to the system.

The point of contact of the envelope with any curve of the system is characterized by the relation

$$\theta-\tilde{\omega}=\text{const.}$$

Thus the angular distance of the point of contact from a line which is fixed relatively to the curve is the same for all the curves. It follows that the locus of any point of the curve is also an equiangular spiral with the same angle but with a scale which depends on the position of the chosen point. In particular, this is true of the apses.

In the actual work of finding the envelope we have to solve the equation

$$\frac{\delta}{\delta\theta} f(\theta-\tilde{\omega}, e)=k ,$$

in order to obtain $\tilde{\omega}$ in terms of θ for substitution in

$$r=a_0 \text{ exp. } k\tilde{\omega} \,.\, f(\theta-\tilde{\omega}, e) .$$

Thus while the angle of the spiral depends solely on the assumed relation between a, $\tilde{\omega}$, its scale depends also on e.

20. In applying these results to the orbits, it is to be noticed that $dr/dt \div rd\theta/dt$ is the same constant in the spiral and the orbits. Since $rd\theta/dt$ has the same sign in all the orbits, it follows that dr/dt has the same sign throughout the spiral arms, that is, the motion along the arms will always be outward.

If each orbit is symmetrical across the center, that is, if the equation is unchanged when we put $\pi+\theta$ for θ, the spiral will consist of two similar and opposite arms. The same result will hold if the orbits are in symmetrical and opposite pairs. In this connection, it is interesting to notice that the double arms may be the ultimate result of ejection from a single place on the nucleus as well as from opposite ends of a diameter.

21. In general, the shape of an orbit will depend on a as well as on e, and the foregoing investigation ignores this fact. But it is applicable to elliptic motion under either of the forces μr or Mr^{-2}, though not to their combined action. What is assumed, however, in applying the results to the spiral nebulae is that the change of shape due to any cause is small compared with the changes of scale and orientation. The assumed equiangular spiral is only an approximation to the actual form. There will be departures from this form if the orbits change in shape, but if de/e is small compared with da/a, the departures will be small.

Further, if the equiangular spiral $r=b \exp. k\theta$ be approximately satisfied in the spiral for a limited range only of values of θ, say from $\theta=0$ to $\theta=\alpha$, we can equally well take the equation

$$r=b_0+b_1\theta+b_2\theta^2+\ldots.$$

to represent the arm, provided $k\alpha$ is sufficiently small; if $k\alpha<0.5$, the third term of the expansion of the expotential form of r is less than 0.13 b. Thus the equiangular form can be approximately maintained over a long range if k is very small, that is, if the arms are nearly circular, or if α is not very great, that is, through a small range of angle about the nucleus. Similar statements, of course, apply to the relation between a, $\tilde{\omega}$.

This latter assumed relation probably depends mainly on the mode of formation of the nebula. But it must be also involved in the changes which the orbits have undergone since the initial stage, and the problem in perturbations which this produces is a complicated one.

22. An attempt to test the envelope hypothesis was made by plotting the points for which v_r has a given value in the case of M 33. According to section 19 these points should lie on a spiral with the same angle as the arms. In this nebula, the larger radial positive velocities appear to be near the points of contact and the number of the measured larger negative radial velocities is too small, so the only available values appeared to be those for which v_r is near zero. I plotted the forty-two points for which $v_r=0\rlap{.}''000$, $\pm 0\rlap{.}''001$, and obtained at some distance from the nucleus indications which could be interpreted as spiral loci. Of the four arms required by central ellipses, portions of three were easily seen, the fourth being doubtful. But owing to the large relative probable error of a single measure and the inequalities of the distribution of the measures but little weight can be attached to such a test.

YALE UNIVERSITY
June 18, 1924

THE MEAN COLOR-INDEX OF STARS OF DIFFERENT APPARENT MAGNITUDES[1]

By FREDERICK H. SEARES

ABSTRACT

Variation of mean color-index with magnitude.—The increase in mean color with magnitude for stars of all galactic latitudes together has been determined from the data in the *Revised Harvard Photometry*, the *Draper Catalogue*, the *Göttingen Aktinometrie*, the *Yerkes Actinometry*, and the magnitudes and colors of stars near the North Pole. The results to the seventeenth magnitude are expressed by the linear relations

(Stars grouped according to visual magnitude) $C_v = +0.50 + 0.029 m_v$ (6)

(Stars grouped according to photographic magnitude) $C_p = -0.18 + 0.071 m_p$. (7)

Kreiken's measures of effective wave-lengths of stars in the Milky Way (mean lat. $= 5°$) also show an increase in mean color with increasing magnitude, but the absolute values and the rate of change are smaller than those defined by (6) and (7).

Variation in mean color with galactic latitude.—Data from the *Revised Harvard Photometry*, the *Draper Catalogue*, and the *Göttingen Aktinometrie* give values for mean magnitude 7.4, which are illustrated by the curves in Figure 2. The pronounced galactic concentration of blue stars is plainly seen in the relatively small mean color-indices of the stars in low latitudes. A few fragmentary results for latitudes less than 30° are also available for stars of the fourteenth magnitude.

Transformation of star counts for a grouping according to visual magnitude into a grouping according to photographic magnitude, and vice versa.—This transformation is of much importance in combining data derived by visual (or photovisual) and photographic methods for a study of the distribution of stars over the face of the sky. It can be effected by applying a color-correction to the magnitudes, that required being approximately the mean of the values of C_v and C_p. For all latitudes together, the result from (6) and (7) is

$$C = +0.16 + 0.050\,m. \qquad (10)$$

This is to be added in transforming from a visual to a photographic grouping, and subtracted for the reverse transformation.

The average color of the stars of different apparent magnitudes is an item of importance in statistical discussions of stellar distribution. Thus star counts have been based on both photographic and photovisual scales of magnitude. To combine these results, as must be done in deriving the surface distribution of stars over the sky, the use of mean colors is necessary, since the colors of individual stars are mostly unknown. Further, the time is approaching when the determination of luminosity curves for the different spectral types can be undertaken. A knowledge of mean color will give a valuable control on the results of this difficult investigation, for naturally the adopted frequency curves must provide a mixture of spectral types which will reproduce the observed colors.

[1] *Contributions from the Mount Wilson Observatory*, No. 287.

It should be emphasized at the outset that the relative frequency of different colors, and hence the mean color, depends upon the grouping of the stars, whether according to photographic or visual magnitude. It is also important to note that the means for neither method of grouping can be used directly to transform the data for one grouping into those of the other. This question will be considered in detail later.

The material now available is sufficient for the derivation of the mean color-index over a considerable range in brightness for all galactic latitudes together, and for a general indication of the variation of mean color with distance from the Milky Way.

The results are affected by several uncertainties, but none of these is of serious consequence. For example, some of the magnitude scales to which the counts are referred require small corrections to reduce them to a homogeneous system; but since the change in mean color with magnitude is slow, the influence of scale error is very small. Further, the colors in many instances must be inferred from the spectral types; but color, for the later types at least, depends on absolute magnitude as well as spectrum. The mean absolute magnitude of each apparent magnitude is known and could have been used in deriving the color-indices, but for simplicity it has been assumed that stars brighter than about 6.5 are all of zero absolute magnitude, while the fainter stars are supposed to have the mean intrinsic brightness of the giants and dwarfs. Again, the results suffer somewhat from systematic errors of spectral classification and from incompleteness of the counts of stars near the limiting magnitudes of the various series. Finally, the stars of faint apparent magnitudes are much restricted in their range in galactic latitude.

For the discussion of the brighter stars the Harvard data are fundamental. These have been collected in spectral groups for which the median spectral types are as shown in Table I:

TABLE I

SPECTRAL GROUPS AND COLOR-INDEX

Harvard group	B	A	F	G	K	M
Median spectral class	B2	Ao	Fo	F8	Ko	M
Color-index, $m<6.5$	−0.26	0.00	+0.38	+0.83	+1.48	+1.88
Color-index, $m>6.5$	−0.26	0.00	+0.38	+0.76	+1.25	+1.80

The color-indices in the last two lines are on the international system, and were taken from *Mount Wilson Contribution* No. 226, Table XII,[1] in accordance with the convention mentioned above. The details for the various sources of data are as follows:

Revised Harvard Photometry.—Tables XVII and XVIII of *Harvard Annals*, **64**, IV, show in detail the distribution of nearly 9000 stars according to visual magnitude and spectral type. Results are given separately for four regions of the sky, designated by N,S,P,M, respectively. The first two are outside the Milky Way, to the north and the south; the third is partly, and the fourth mainly, in the Milky Way. These data are condensed into the totals shown in the upper half of Table II and in Table III, the former of which refers to half-magnitude intervals for the whole sky, the latter to the totals in the separate regions for all magnitudes together. The column heading indicates the median magnitude, except that the second column refers to stars brighter than $m=2.25$, with a mean of about 1.6.

The mean color-indices, C_v, for a grouping according to visual magnitude, are found at once with the aid of the data in Table I. The mean values in Table III, for all magnitudes together, correspond to a mean magnitude of 5.7.

To obtain the results for a grouping according to photographic magnitude, it is to be noted that the counts for any line and column in the upper part of Table II refer to the photographic magnitude $m+C$, where m is the column heading and C the color-index for the spectral group in question. Thus for group G, the numbers 2, 12, 12, etc., beginning with the third column, correspond to median photographic magnitudes 3.33, 3.83, 4.33, etc., respectively. By plotting numbers, or better, the logarithms of the numbers, against photographic magnitudes, the totals in each spectral group for the median photographic magnitudes 2.5, 3.0, 3.5, etc., are readily found. These are collected in the lower half of Table II; the mean color-indices, C_p, for the photographic grouping, appear in the last line.

For the late spectral types the limits of photographic magnitude for the different groups are shifted from one to two magnitudes below the visual limits, while for the B stars there is a small shift

[1] *Astrophysical Journal*, **55**, 198, 1922.

in the opposite direction. The full utilization of the material, therefore, requires some extrapolation of the curves; but this can be done with little uncertainty, for when plotted in logarithmic form the curves are very smooth and nearly linear. This extrapola-

TABLE II

MEAN COLOR-INDEX FROM *Revised Harvard Photometry*
(All Galactic Latitudes Together)

Sp.	1.6	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5
B	18	9	28	27	51	103	164	170	140	105	......	......
A	12	10	10	25	41	94	141	541	959	872	......	......
F	4	4	8	10	17	46	101	186	345	376	......	......
G	6	2	12	12	32	47	94	127	277	323	......	......
K	8	6	21	44	91	129	239	419	762	821	......	......
M	7	5	9	12	17	39	87	120	169	172	......	......
Totals	55	36	88	130	249	458	876	1563	2658	2669	......	......
C_s	0.49	0.53	0.61	0.73	0.75	0.64	0.68	0.63	0.66	0.72	......	......
Res	+0.06	+0.04	−0.02	−0.13	−0.13	−0.01	−0.04	+0.03	+0.01	−0.03	......	......
B	......	13	23	42	74	129	166	158	126	87	58	40
A	......	7	13	25	50	100	195	398	724	1259	1778	2399
F	......	1	4	7	14	27	55	112	209	372	575	794
G	......	1	2	4	9	19	35	62	102	182	295	427
K	......	1	2.2	4.5	9.3	20	41	81	155	269	468	776
M	......	0.3	0.5	10	2.2	4	8	15	28	48	79	126
Totals	......	23.3	44.7	83.5	148.5	299	500	826	1344	2217	3253	4562
C_p	......	0.00	0.03	0.04	0.08	0.09	0.17	0.24	0.31	0.34	0.40	0.45
Res	......	0.00	0.00	+0.03	+0.02	+0.05	+0.01	−0.03	−0.06	−0.06	−0.08	−0.10

TABLE III

COLOR-INDEX AND GALACTIC LATITUDE FROM *Revised Harvard Photometry*

Sp.	N	S	P	M	All
B	44	97	197	483	821
A	525	381	787	1062	2755
F	210	254	292	341	1097
G	207	199	224	302	932
K	533	594	616	797	2540
M	137	147	149	204	637
Totals	1656	1672	2265	3189	8782
C_s	0.78	0.83	0.63	0.57	0.675 ($m=5.7$)
Lat	+51°.2	−50°.2	21°.9	8°.8	....
C_p	0.38		0.29	0.22	.... ($m=5.5$)

tion accounts for the extension of the limit for the photographic data to 7.5. The weight for the last two values of C_p is naturally low, and there is probably a small systematic error arising from the fact that the numbers for the B stars are obviously incomplete, since they decrease from about 5.5 on.

The values of C_p in the last line of Table III were found by a transformation similar to that just described.

Henry Draper Catalogue.—In Table III of *Harvard Circular* 226, Shapley has given the relative frequencies of 98,675 stars brighter than visual magnitude 8.75 as derived from counts based on the *Henry Draper Catalogue*. The form of the table is similar to Table II above, except that frequencies instead of numbers are given. The mean values of C_v found from these data with the aid of the last line of Table I are in the last line but one of Table IV.

To find the values of C_p the data of Table II of *Harvard Circular* 226 were referred to the photographic scale by the method used

TABLE IV

MEAN COLOR-INDEX FROM *Henry Draper Catalogue*

(All Galactic Latitudes Together)

Sp.	5.4	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
B......	20.8	10.5	7.1	5.3	3.3	2.5	1.5	1.0	0.6
A......	50.3	52.7	53.2	48.5	52.0	49.8	41.8	39.5	37.7
F......	10.4	13.0	13.3	15.2	13.6	13.9	14.2	11.1	7.8
G......	5.5	7.8	9.5	11.2	11.9	13.6	18.4	22.0	27.1
K......	11.1	13.9	15.0	17.6	17.0	18.2	21.8	23.5	23.7
M......	2.0	2.0	2.0	2.1	2.1	2.1	2.3	2.9	3.0
C_p......	0.20	0.29	0.33	0.39	0.38	0.42	0.50	0.58	0.61
Res.....	.00	− .01	− .01	− .04	+ .03	.00	− .04	−0.08	−0.08
C_v......	.61	.66	.68	.75	.78	.76	.74	....	....
Res.....	+0.05	+0.03	+0.02	−0.03	−0.05	−0.01	+0.02	....	....

in the case of the *Revised Harvard Photometry*. The results represent the total numbers of stars for each spectral group to successive limits of photographic magnitude. The numbers in each half-magnitude interval found by differencing the totals were reduced to relative frequencies with the results shown in Table IV. The values of C_p, calculated as before, are in the fourth line from the bottom of the table.

The limiting magnitude to which the *Henry Draper Catalogue* is complete is stated by Shapley to be about 8.25 for the northern sky and 8.75 for the southern. For the present purpose incompleteness in the fainter magnitudes given in Table IV is of no consequence, provided the selection of spectral types is representative. Of this

we cannot be sure, but in any case the effect on the mean color is probably not serious.

The frequencies of the spectral classes in different galactic latitudes are not yet available for the *Draper Catalogue* in numerical form, but Shapley has given them diagrammatically,[1] for all magnitudes together, to the visual limit 8.25. Frequencies scaled from this diagram and combined with the values of the color from the last line of Table I lead to the following mean colors:

Gal. lat.	0°	10°	30°	45°	60°	75°	90°	(1)
C_v	0.51	0.62	0.79	0.86	0.94	0.94	0.93	
Smoothed	0.51	0.62	0.78	0.86	0.91	0.94	0.94	

The number of stars involved is 58,508,[2] and their mean magnitude is 7.4.

Göttingen Aktinometrie.—Counts of the stars in this catalogue for half-magnitude intervals of Potsdam visual magnitude and various intervals of the color-index J_p were made for another purpose some years ago. These are collected in Table V, for which the magnitude headings, as usual, are median values. The mean values of J_p within the intervals used were found from the data in Table 29*a* of the *Aktinometrie* (Part B, p. 34), and reduced to the international system by adding +0.29. This reduction constant was found by comparing the data in the first two columns of Table 23 of the *Aktinometrie* with the Mount Wilson color-indices for stars of absolute magnitude zero.

The resulting values of C_v, in the fourth line from the bottom of Table V, are seriously influenced by selection. The general increase in mean color with increasing magnitude is masked, if not actually reversed. The values for the bright stars are too large because the photographic images of early-type stars were oftentimes too dense for measurement, while late-type stars of the same visual magnitude were easily measurable. For faint stars, on the other hand, the selection operates in the opposite direction. The value for the middle of the series, $C_v=0.71$ for $m=6.25$, is probably the only one that can be trusted.

The difficulty should disappear, however, when the counts are

[1] *Proceedings of the American Academy of Arts and Sciences*, **59**, 227, 1924.

[2] *Harvard Circular* 226, Table II.

reduced to a photographic grouping by the method already used. The values of C_p thus found are in the last line but one of Table V. As usual, the first and last values are less reliable because of extrapolation of the transformation curves.

TABLE V

MEAN COLOR-INDEX FROM *Göttingen Aktinometrie*
(All Galactic Latitudes Together)

Limits J_p	Mean Color Int. System	4.25	4.75	5.25	5.75	6.25	6.75	7.25	7.75	8.25
<−0.32	−0.15	......	15	18	43	80	126	129	60	......
−0.32 to 0.00	+ .13	4	16	29	41	84	145	217	159	......
.00 to + .34	.41	8	8	19	41	54	109	182	148	......
+0.35 to +0.99	0.96	10	21	28	55	117	215	264	129	......
+1.00 to +1.90	1.57	10	21	42	68	117	207	189	54	......
>+1.90	+2.30	......	2	1	2	3	2	2	......	......
Totals	....	32	83	137	250	455	804	983	550	......
C_v	....	0.91	0.73	0.76	0.72	0.71	0.72	0.65	0.51	......
Res	....	− .29	− .09	− .11	− .05	− .03	− .02	+ .06	+ .21	......
C_p	....	.29	.27	.27	.30	.31	.33	.35	.41	0.46
Res	....	−0.17	−0.11	−0.08	−0.07	−0.05	−0.03	−0.02	−0.04	−0.05

Although the *Aktinometrie* includes only the zone 0°–20° in declination, it gives valuable information as to the variation of color with galactic latitude. Tables 29*a*, 29*b*, and 30 contain data bearing on the question, which, it is stated, refer to the stars between 4.50 and 6.99 of *photographic* magnitude. This, however, seems to be an error: The mean values of the color are in agreement with those found from other sources for a grouping according to visual magnitude, and a recount for the data given in Table 30, based on a grouping according to Göttingen photographic magnitude, shows clearly that this is the case.

TABLE VI

COLOR AND GALACTIC LATITUDE, *Göttingen Aktinometrie*, $m=6.3$

R.A.	Gal. Lat.	C_v	C_p	R.A.	Gal. Lat.	C_v	C_p
0^h5	−52°	0.84	0.40	12^h5	+72°	0.85	0.38
2.5	44	.64	.30	14.5	59	.88	.42
4.5	−23	.56	.22	16.5	34	.85	.43
6.5	+ 2	.62	.17	18.5	+ 7	.72	.25
8.5	28	.78	.31	20.5	−18	.93	.36
10.5	+54	0.81	0.37	22.5	−40	0.79	0.41

The third and seventh columns of Table VI show the results from Table 30 of the *Aktinometrie*, which are here given their correct

designation C_v. The fourth and eighth columns give the values of C_p based on the recount for intervals of photographic magnitude. Both series have been reduced to the international system of color and refer to the mean magnitude 6.3. The galactic latitudes are for the centers of the areas over which the colors have been averaged. The smoothed results are

$$\left.\begin{array}{lccccccc} \text{Gal. lat.} \ldots\ldots & 0^\circ & 10^\circ & 30^\circ & 45^\circ & 60^\circ & 75^\circ & 90^\circ \\ C_v \ldots\ldots & (0.66) & (0.70) & (0.76) & 0.80 & 0.85 & 0.88 & 0.90 \\ C_p \ldots\ldots & 0.19 & 0.24 & 0.32 & 0.37 & 0.40 & 0.42 & 0.44 \end{array}\right\} \qquad (2)$$

The values of C_v, like those in Table V, are affected by the selection in the data described above. The exclusion of the bright early-type stars increases the mean color in low galactic latitudes. In high latitudes, on the other hand, early-type stars occur but rarely. The compensating influence arising from the omission of faint late-type stars probably does not enter to any appreciable degree, because the lower limit is visual magnitude 6.99 and the totals in Table V show that to this limit they are nearly if not quite complete. A comparison of the second line of (2) with the last line of (1) supports this view. From 45° to 90° the differences are practically constant, with a mean value of 0.055. The difference to be expected because of the fact that the data from the *Draper Catalogue* are for mean magnitude 7.4 while those of the *Aktinometrie* are for 6.3 is 0.032 mag. The agreement for the higher latitudes is therefore within the uncertainty, but for low latitudes the results for C_v in (2) are certainly too large. The values of C_p, on the other hand, should be reliable for all latitudes.

Yerkes Actinometry.—The number of stars in the catalogue is not large, and the range in galactic latitude is small. It will be sufficient to quote here the result derived by Kapteyn,[1] who found for the stars of visual magnitude 6–7

$$C_v = +0.66 \qquad (m = 6.6). \qquad (3)$$

North polar stars.—Well-determined color-indices are available[2] to photovisual magnitude 17.4. Mean values are given in Table VII.

[1] *Mt. Wilson Contr.*, No. 83; *Astrophysical Journal*, **40**, 187, 1914. eq. (16).

[2] *Transactions International Astronomical Union*, **1**, 69, 1922.

Stars brighter than about the twelfth magnitude are omitted; they are few in number and many of them are selected because of small color-index. The galactic latitude is 28°.

Since the photographic magnitudes extend to about magnitude 20, a grouping according to photovisual magnitude should be free from the influence of selection. This is not true, however, when the data are grouped according to photographic magnitude. For example, color-indices are available for stars as faint as photographic magnitude 19.1, but these stars are all very red. Stars of this photographic brightness having moderate color-indices do not occur among the data because their photovisual magnitudes are below the limit 17.4. The last value of C_p in Table VII is certainly too

TABLE VII

MEAN COLOR-INDICES FOR NORTH POLAR STARS

Pv. Mag.	C_v	Res.	No. Stars	Pg. Mag.	C_p	Res.	No. Stars
12.2	0.86	−0.04	18	12.8	0.77	−0.04	25
13.7	.90	.00	28	14.6	.81	+ .05	27
14.5	.83	+ .09	25	15.4	.98	− .07	26
15.4	0.94	+ .01	16	16.3	0.93	+ .05	12
17.1	1.30	−0.30	17	18.3	(1.29)	(−0.17)	19

large; but there is no reason to suspect any disturbance of the last C_v beyond that arising from the small number of stars.

Field of S Cygni.—The colors of about 400 stars between photovisual magnitudes 12 and 17 have been measured at Mount Wilson. The increase in the minimum color-index appears, as it did in the case of provisional values for about 200 stars in this region which were illustrated in *Contribution* No. 81;[1] but the customary increase in mean color is masked by a large percentage of red stars between the twelfth and fourteenth magnitudes. Moreover, the magnitude scales are not wholly satisfactory. It seems best, therefore, to use only mean values for the entire series. The results are

$$\left.\begin{array}{l} C_v=0.70\ (m=13.9) \\ C_p=0.70\ (m=14.3) \\ \text{Gal. lat.}=13^\circ \end{array}\right\} \qquad (4)$$

[1] *Astrophysical Journal*, **39**, 361. 1914.

Colors of faint stars in the Milky Way.[1]—The color-indices of about 3350 stars in eight different regions of the Milky Way were determined by Kreiken from objective grating photographs made by Hertzsprung with the 60-inch reflector at Mount Wilson in 1912. The galactic latitudes for the separate regions range from $-8°$ to $+12°$, with a mean of $5°$. The colors are on the international system, and were grouped by Kreiken according to intervals of photographic magnitude. Table 38 of his paper shows that the systematic differences in the mean colors from region to region are

TABLE VIII

NUMBERS AND MEAN COLORS OF STARS IN MILKY WAY (KREIKEN)

C. I.	10	11	12	13	14	15
−0.6	.....	.....	.....	.....	2	5
.4	1	2	.....	1	2	12
− .2	3	12	4	14	12	22
.0	3	14	16	60	58	48
+ .2	9	29	53	185	197	161
.4	6	20	41	121	222	210
.6	3	8	17	51	156	235
0.8	1	5	11	35	65	180
1.0	2	8	10	43	82	141
1.2	3	7	9	37	76	119
1.4	2	3	3	32	32	70
1.6	1	2	1	11	21	36
1.8	.....	.....	.....	1	3	20
+2.0	.....	.....	.....	.....	1	10
Totals	34	110	165	591	929	1269
C_p	0.46	0.39	0.43	0.49	0.57	0.70
C_v	0.67	0.68	0.69	0.70	0.72	0.78

not large. The counts have therefore been combined, and, in a somewhat abbreviated form (3098 stars), are given in Table VIII. With the exception of the first value of C_p, which depends on a small number of stars, the mean color-index shows the characteristic increase. The results for C_v were obtained by the usual transformation.

Harvard Standard Regions.—A discussion of about 800 stars in these regions by Miss Leavitt has been given in *Harvard Circular* 230. The counts according to photographic magnitude in Table I of that

[1] E. A. Kreiken, *On the Colour of the Faint Stars in the Milky Way and the Distance of the Scutum Group.* This publication bears neither date nor imprint, but seems to have been issued from Groningen.

publication have been combined and appear in Table IX below. The spectral types are the mean values given by Miss Leavitt. The corresponding color-indices in the second column of the table are Mount Wilson values and represent the mean for giant and dwarf stars. The increase in the resulting values of C_p is abnormal,

TABLE IX

MEAN COLOR-INDEX FOR HARVARD STANDARD REGIONS

Sp.	C. I.	7	8	9	10	11
B5	−0.18	7	11	2	1	.....
A1	+ .04	41	63	59	27	3
A9	.34	21	29	33	9	.....
F9	.75	22	60	50	48	9
G9	1.20	26	66	68	80	30
K6	+1.60	2	6	14	10	5
Totals	.....	119	235	226	175	47
C_p	.....	0.49	0.61	0.68	0.87	1.08
Res.	.....	−0.17	−0.22	−0.22	−0.34	−0.40

and a comparison with the other data shows that the entire series is systematically too large by about 0.3 mag. The counts are obviously incomplete, and apparently the selection has been such as to favor the stars of high color. When grouped according to the four sections of the sky designated by N, S, P, and M, the means for all magnitudes together ($m=8.7$) are

$$\left.\begin{array}{lcccc} & N & S & P & M \\ \text{Gal. lat.} & +51.2 & -50.2 & 21.9 & 8.8 \\ C_p & 0.85 & 0.79 & 0.73 & 0.42 \end{array}\right\} \qquad (5)$$

COMBINATION OF DATA

Change in mean color-index with magnitude.—The results for all galactic latitudes together derived from the *Revised Harvard Photometry*, the *Draper Catalogue*, and the *Göttingen Aktinometrie* (Tables II, IV, V) are shown graphically in Figure 1. The Mount Wilson values for stars near the North Pole (Table VII) are also included. The galactic latitude of 28° is not far from the mean for all the stars of moderate brightness,[1] and at present these data

[1] Because of the change in galactic concentration with increasing magnitude, the mean latitude also varies with the magnitude of the stars considered.

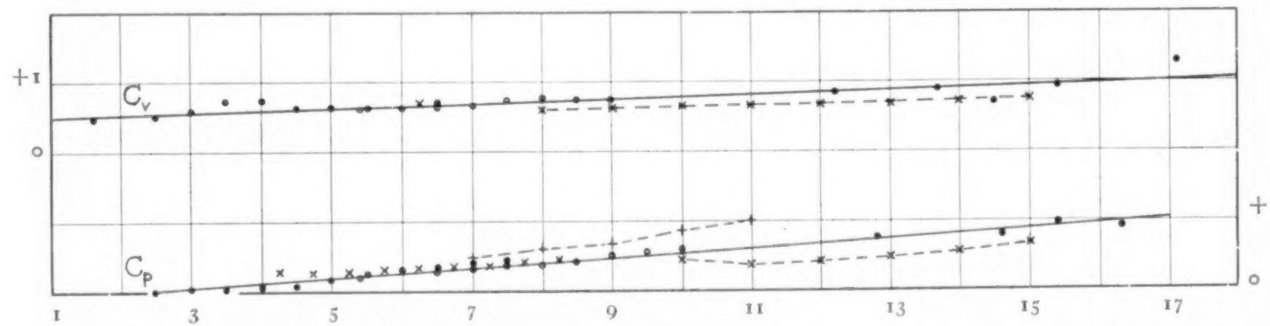


FIG. 1.—Mean color-index and apparent magnitude, all galactic latitudes together. C_v, stars grouped according to visual or photovisual magnitudes; C_p according to photographic magnitude. Points: data from *Revised Harvard Photometry*, and, for faint stars, Mount Wilson measures at the Pole; circles: *Henry Draper Catalogue;* crosses on full-line curve: *Göttingen Aktinometrie.* The dotted curves for faint stars are results from Kreiken's measures in the Milky Way (mean lat. $=5°$), and show the influence of the concentration of blue stars in this region. The discordant values for C_p between $m=7$ and 11 are from the Harvard Standard Regions and show the effect of selection in the data.

provide the only means available for extending the mean results for all latitudes to the faint stars. Kreiken's measures of fields in mean latitude 5° are so close to the galactic circle that they supply only a rough control on the general accuracy of the results.

An inspection of Figure 1 shows that the values of both C_v and C_p as functions of the magnitude are well represented by simple linear relations. Those adopted are

$$\text{All latitudes together}\begin{cases} C_v=+0.50+0.029m_v & (6)\\ C_p=-0.18+0.071m_p. & (7)\end{cases}$$

The corrections to the tabular values necessary to reduce them to those defined by (6) and (7) are given in the various tables opposite the designation "Res." The influence of selection upon the values of C_v for the *Göttingen Aktinometrie* (Table V) is clearly evident, and the large systematic discordance for the Harvard Standard Regions is shown in the last line of Table IX. Otherwise, the residuals are well within the inherent uncertainties.

Variation of mean color with galactic latitude.—Most of the data bearing on the change in color with latitude refer to stars between mean magnitudes 5.7 and 7.4. It cannot be supposed that the coefficients of m in (6) and (7) are valid for all galactic latitudes; but no serious error will be introduced by using them to calculate the changes in C for an interval of one or two magnitudes. In this way the values of C_v for the *Harvard Photometry* in Table III ($m=5.7$) and for the *Göttingen Aktinometrie* in (2) ($m=6.3$) can be reduced to $m=7.4$ for comparison with the results from the *Draper Catalogue* given in (1). The corrections are +0.05 and +0.03, respectively. The agreement of the three series of results is shown graphically in the upper part of Figure 2. With the exception of the first two points for the *Göttingen Aktinometrie*, which have been discussed above, the deviations from the curve for the *Draper Catalogue* are small, and for the present it will be sufficient to accept this curve as a reasonably good approximation.

The lower part of Figure 2 shows the agreement of the values of C_p from the *Harvard Photometry* (last line of Table III) with those from the *Aktinometrie*, given in (2). The systematic difference is 0.02 mag., which again is well within the uncertainty of the data.

Reduced to $m=14.0$, the results for S Cygni given in (4) are shown in the third column of Table X. The values for the same magnitude found from Kreiken's data (Table VIII) and from the

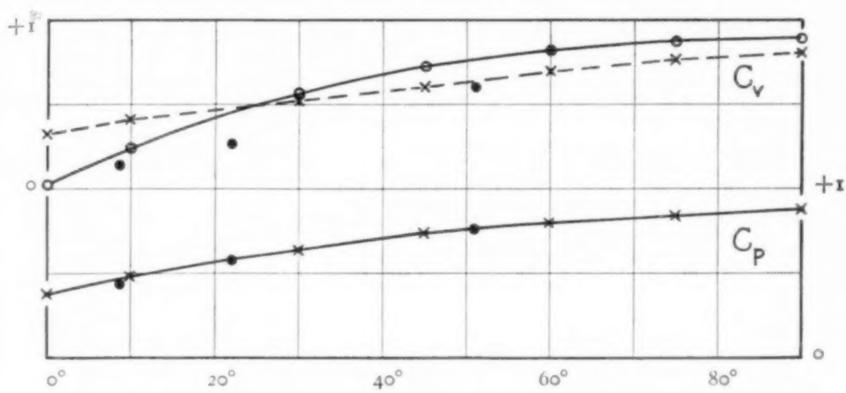


FIG. 2.—Mean color-index and galactic latitude for stars grouped according to visual (C_v, $m=7.4$) and photographic (C_p, $m=6.3$) magnitude. Points: *Revised Harvard Photometry;* circles: *Henry Draper Catalogue;* crosses: *Göttingen Aktinometrie.*

polar stars by (6) and (7) are in the second and fourth columns of this table. C_v for S Cygni is the only discordant value. By comparing Kreiken's values for $m=14$ with $C_v=0.56$ and $C_p=0.20$,

TABLE X

MEAN COLORS FOR $m=14.0$

	Kreiken, Milky Way	S Cygni	N. Polar Stars
Lat.	5°	13°	28°
C_v	0.72	0.70	0.91
C_p	0.57	0.68	0.81

read from the curves in Figure 2 for latitude 5°, and assuming a linear increase in color with m, we find

$$\text{Gal. lat.}=5^\circ \begin{cases} C_v=+0.38+0.024m_v & (8)\\ C_p=-0.10+0.048m_p . & (9)\end{cases}$$

The coefficients in these relations are in general agreement with the variation in color shown by Kreiken's measures in the interval $m=10$ to 15. The relatively slow increase in the mean color in the Milky Way indicated by a comparison of (8) and (9) with (6)

and (7) is in accordance with what we know of the distribution of colors among bright and faint stars in different parts of the sky.

Transformation of star counts for a grouping according to visual magnitude into a grouping according to photographic magnitude, and vice versa.—A simple illustration shows that the values of C_v and C_p found above cannot be used directly for this transformation. The increase with magnitude in the number of stars in a constant interval of magnitude is nearly that of a geometrical progression. Suppose this to be strictly the case, and, further, suppose that we have only two classes of stars, I and II, with color-indices of 0 and 1 mag., respectively. Let the numbers for a grouping according to visual magnitudes be as shown in the second, third, and fourth lines of Table XI. With the adopted color-indices, the numbers

TABLE XI

Median mag.	4	5	6	7	8
No. I_v	....	2	4	8	16
No. II_v	2	4	8	16	32
Total $=N_v$	....	6	12	24	48
No. I_p	....	2	4	8	16
No. II_p	....	2	4	8	16
Total $=N_p$	....	4	8	16	32

for a photographic grouping will then be those given in the last three lines of the table.

The mean color-indices C_v and C_p are in both cases constant and equal, respectively, to 0.67 and 0.50. The mean photographic magnitudes of the groups N_v are therefore

$$p' = \dots . \ 5.67, 6.67, 7.67 \ \dots ,$$

while the mean visual magnitudes of the groups N_p are

$$v' = \dots . \ 4.50, 5.50, 6.50 \ \dots .$$

We cannot, however, take the sequences p' and N_v and by interpolation arrive at the true values of N_p for photographic magnitude = 5, 6, and 7. The result of such an interpolation is 3.77, 7.55, and 15.10, whereas the correct values, as shown in the last line of Table XI, are 4, 8, and 16. Nor can we use the sequences v' and N_p and by interpolation obtain correct values of N_v, that is, of the

totals for a grouping according to visual magnitude. In each case a statistical error enters; these errors are opposite in sign, and, for the data of the illustration, equal in percentage amount.

In the absence of definite knowledge of the frequencies of the various spectral types for different magnitudes the statistical errors are difficult to determine, and it is simpler to approach the question by comparing the values of N_v and N_p and ask what photographic magnitudes, in a grouping according to this magnitude, will correspond to the totals N_v (or vice versa). This comparison of totals, or better, a comparison of the values of log N_v and log N_p, leads to the following sequences of corresponding magnitudes:

Visual................	5	6	7
Photographic.........	5.585	6.585	7.585

The meaning of the sequences is that if, in a grouping according to visual magnitude, a total of N stars corresponds to the median magnitude m, then, in a grouping according to photographic magnitude, the total N will correspond to median magnitude $m+0.585$. Hence, the color-index which must be applied to the magnitudes in order to transform the totals grouped according to visual magnitude into the corresponding grouping according to photographic magnitude is 0.585. The result for the reverse transformation is the same. From conditions of symmetry we should expect this value to be sensibly equal to the mean of $C_v=0.67$ and $C_p=0.50$, which, in fact, is the case.

This suggests a practical method of procedure. The relations between the actual counts are not as simple as those of the illustration. Nevertheless, only small errors of the second order will be introduced by using the mean values of C_p and C_v found above, which for the case of all galactic latitudes together are defined by equations (6) and (7). For this general case the mean color-index for the transformation of star counts referred to visual scale into counts referred to the photographic scale, or vice versa, is therefore

$$C=+0.16+0.050\,m. \tag{10}$$

This relation is approximate, but within the limits of error of the existing data.

Mount Wilson Observatory
September 1924

INVESTIGATIONS ON PROPER MOTION

ELEVENTH PAPER: THE PROPER MOTION OF MESSIER 13 AND ITS INTERNAL MOTION[1]

By ADRIAAN VAN MAANEN

ABSTRACT

Proper motions and internal motion of M 13.—Two pairs of photographs taken with the 80-foot focus arrangement of the 60-inch Mount Wilson reflector, with intervals of eleven and nine years, respectively, were measured for proper motion. The results indicate motions of the cluster as a whole amounting to $0''.001$ in each co-ordinate, while the internal motions are less than $0''.001$ annually.

The results are therefore negative, but they afford a good confirmation of Shapley's estimate of the distance of this object, and, furthermore, show that the internal motions found in several spiral nebulae cannot be due to the 60-inch reflector or to the measuring instrument used.

The unexpected discovery of internal motions in spiral nebulae by the writer made it seem desirable to try measures of a globular cluster as soon as the interval of time between old and new plates had become such as to make measurable displacements a possibility. Before the measurement of M 101 it was generally supposed that the spirals were located at enormous distances. The evidence for this belief, however, was principally the appearance of the many novae in these objects; and its inconclusiveness was pointed out in *Mount Wilson Contribution*, No. 243 (pp. 6 and 7).

Although the enormous distances of the globular clusters as derived by Shapley are based on considerably more substantial data than were those for the spiral nebulae, it seemed worth while to attempt to discover whether measurable motions, either internal or of the clusters as a whole, could be detected.

Several photographs of the great cluster in Hercules, M 13, were obtained soon after the 80-foot focus arrangement of the 60-inch Mount Wilson reflector had been made available for direct photography, and many other photographs of this and other clusters were taken later. Those listed in Table I were selected for measurement. The plates were combined into two pairs, 3878–5 and 3728–119, and measured in four positions each with the new stereo-comparator in the manner used for the later measures of the spirals.

[1] *Contributions from the Mount Wilson Observatory*, No. 284.

Dr. Shapley had indicated which stars might be members of the cluster and which not, the latter to be used for comparison purposes. With his help fifty comparison stars were selected, while sixty-two other stars were measured, which, with considerable probability, might be accepted as members of the cluster. His selection was based on the distance from the center, the magnitude, and the color-index of the individual stars.

The results of the measures are collected in Table II. When available, Ludendorff's number has been used; otherwise the stars are designated by the letters *a . . . v*. The first fifty in the list are the comparison stars. The magnitudes in the second column are the photographic magnitudes derived by Dr. Shapley; the co-ordinates in the third and fourth columns are given in seconds of

TABLE I

Plate No.	Date	Exp.	Plate	H.A. at Middle of Exp.	Quality
5	1913, May 26	30^m	Seed 27	$-\ 1^{\circ}.2$	g
119	1914, April 18	30	Seed 27	-11.7	g
3728	1923, June 8	30	Seed 23	-10.5	g
3878	1924, May 30	$31\frac{1}{4}$	Seed 30	$+18.5$	fg

arc and are taken from Ludendorff's catalogue,[1] except for 1120 and the stars *a . . . v*, which do not appear in that source. For these the co-ordinates were derived from one of the 80-foot focus plates and are accurate within 2 or 3 seconds of arc, which is sufficient for identification purposes. The origin of the co-ordinates is the same as that used by Ludendorff, viz., $\alpha = 16^h38^m6^s$, $\delta = +36^{\circ}19'19''$ (1900).

For the reduction, the two sets of measures of each pair of plates in the two positions in right ascension and declination were combined into single sets. The displacements were then reduced to thousandths of a second of arc per year by multiplying the measured quantities by 0.229 in the case of 3878–5 and 0.276 in the case of 3728–119. The resulting quantities were used as the first members of equations of condition of the form:

$$m_\alpha = a + bx + cy + dx^2 + exy + fy^2 + \mu_\alpha\,.$$

$$m_\delta = a' + b'x + c'y + d'x^2 + c'xy + f'y^2 + \mu_\delta\,.$$

[1] *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, No. 50, 1905.

TABLE II

CO-ORDINATES AND ANNUAL DISPLACEMENTS

(The motions are given in 0".001 as a unit)

No.	Pg. Mag.	x	y	$\mu_{1\alpha}$	$\mu_{2\alpha}$	$\mu_{1\delta}$	$\mu_{2\delta}$	μ_{α}	μ_{δ}
a	14.50	−583"	−288"	− 9	− 6	+ 4	+12	− 8	+ 8
c	14.52	491	−400	+ 4	+ 4	− 6	3	+ 4	− 2
b	13.06	483	+182	17	7	+ 5	+ 9	12	+ 7
e	13.40	415	−144	2	1	+ 3	− 7	2	− 2
d	14.52	411	+346	10	8	−14	−12	9	13
1	14.85	389	+150	6	11	1	+ 2	8	0
f	13.30	364	−371	+ 3	3	− 2	− 2	+ 3	− 2
10	15.14	339	+ 78	− 1	1	+ 6	2	0	+ 2
13	15.50	322	165	0	+ 3	9	− 4	+ 2	2
15	13.06	310	333	−25	−19	+21	+22	−22	+22
19	15.91	298	194	1	+ 5	− 3	− 5	+ 2	− 4
36	16.13	259	+113	10	− 6	+ 1	−18	− 8	8
g	14.95	256	−394	− 7	4	− 4	+ 5	6	0
43	14.52	250	191	+ 5	6	6	− 3	0	4
47	15.89	237	228	+ 3	4	2	+ 3	0	0
h	14.95	176	510	0	4	0	− 2	− 2	1
90	16.22	168	110	−26	13	−18	−25	20	−22
99	16.03	156	− 89	2	7	+13	+11	4	+12
1120	16.17	138	+ 36	13	− 7	−24	− 8	−10	−16
234	16.16	77	222	− 4	+ 4	−13	+ 4	0	− 4
i	15.00	51	+505	+ 3	+ 5	+ 8	+ 8	+ 4	+ 8
331	16.13	45	−249	11	0	11	−11	6	0
j	15.17	6	+478	7	0	2	+ 2	4	2
505	16.16	− 2	264	5	− 6	+ 8	− 2	0	+ 3
557	15.11	+ 10	379	0	+ 7	− 3	− 3	+ 4	− 3
599	16.17	19	+264	3	− 3	−12	+ 4	0	− 4
k	13.82	24	−485	+10	+15	+10	3	+12	+ 6
687	13.76	40	−319	− 3	1	+ 1	+ 5	− 1	+ 3
762	15.95	60	+238	− 3	1	− 1	− 7	− 1	− 4
814	16.20	73	+147	+ 3	4	7	+ 2	+ 4	2
831	16.06	77	−173	7	0	− 4	− 7	4	− 6
1015	16.14	160	253	4	8	+ 5	+ 7	6	+ 6
1019	12.98	163	84	3	6	−15	− 8	4	−12
l	14.34	205	368	+ 8	+ 7	+11	+ 6	+ 8	+ 8
1057	15.86	212	136	− 2	− 5	14	4	− 4	9
1075	15.79	247	−234	+ 6	+ 2	5	12	+ 4	8
1087	13.14	264	+234	− 5	−11	13	+15	− 8	+14
m	14.88	318	456	+ 6	+ 1	+ 3	− 8	+ 4	− 2
1111	16.10	342	+183	− 7	− 5	−19	−10	− 6	−14
1114	14.63	351	−330	+ 8	+12	+ 5	+10	+10	+ 8
1115	15.70	354	+363	− 2	4	6	3	1	4
n	13.90	434	−182	+11	+ 7	10	9	+ 9	10
p	13.14	437	−479	−29	−25	51	57	−27	54
o	14.88	461	+ 75	− 4	− 7	2	0	− 6	1
r	14.09	482	− 58	+ 3	+ 5	+22	+14	+ 4	+18
q	14.33	493	+369	+ 5	+ 3	−25	−22	+ 4	−24
s	14.88	503	−402	− 3	− 2	−110	−111	− 2	−110
t	15.77	553	+392	− 6	3	+ 8	+ 3	− 4	+ 6
u	15.50	557	− 21	+ 1	− 1	16	24	0	20
v	15.28	+590	−181	+ 4	+ 5	+16	+13	+ 4	+14

TABLE II—*Continued*

No.	Pg. Mag.	x	y	$\mu_{1\alpha}$	$\mu_{2\alpha}$	$\mu_{1\delta}$	$\mu_{2\delta}$	μ_{α}	μ_{δ}
2	14.96	−389″	+138″	+ 7	+12	− 4	0	+10	− 2
7	15.04	364	+382	9	+12	3	+12	10	+ 4
17	14.86	307	−273	1	− 2	− 1	− 5	0	− 3
25	15.06	285	+107	+ 6	4	+ 5	− 5	+ 1	0
46	15.57	240	−114	−14	−10	− 3	+ 1	−12	1
49	14.83	235	− 10	+ 4	0	4	− 6	+ 2	5
59	15.08	221	+195	+ 7	+ 6	13	+ 2	+ 6	6
70	13.45	202	+147	− 9	− 5	0	− 2	− 7	1
72	13.74	198	−139	7	9	3	3	8	3
96	13.70	161	+ 10	− 4	7	3	5	− 6	4
118	15.03	143	− 56	+ 6	2	10	2	+ 2	6
158	13.86	118	+113	− 3	5	0	11	− 4	6
222	12.54	82	−103	2	11	− 2	0	6	− 1
246	15.49	74	− 98	− 6	4	+ 8	3	5	+ 2
252	13.84	71	+142	+ 1	1	0	4	0	− 2
299	15.31	56	+ 23	+ 4	7	0	2	2	1
306	var.	54	− 3	− 4	8	+ 6	− 9	6	2
310	15.30	53	−114	+ 3	2	− 2	+ 3	0	0
311	15.50	52	+ 29	−15	7	7	− 4	11	6
313	15.42	51	+ 96	1	4	− 1	0	2	0
323	15.42	47	− 60	10	5	+ 1	1	8	0
325	15.31	46	− 21	− 4	− 5	− 7	4	− 4	6
326	13.76	46	+180	+ 1	+ 3	+ 4	− 8	+ 2	− 2
363	15.18	34	−237	0	0	6	+ 1	0	+ 4
406	15.20	24	87	5	− 4	1	− 8	0	− 4
418	15.46	21	−169	0	− 9	+ 7	5	− 4	+ 1
500	15.44	− 3	+129	2	+ 3	− 2	4	+ 2	− 3
523	15.31	+ 2	42	+ 3	− 5	10	2	− 1	6
559	15.20	10	54	− 7	1	6	1	4	4
641	15.14	29	217	+ 5	5	1	− 2	0	− 2
653	15.18	32	+189	− 2	3	0	+ 4	2	+ 2
686	15.42	40	−135	+ 2	4	− 6	+ 8	1	+ 1
690	15.26	41	+ 56	− 5	0	+ 2	− 7	2	− 2
712	15.36	46	− 69	+ 1	9	− 2	4	− 4	3
737	15.20	54	72	9	6	1	3	+ 2	2
775	14.91	63	−159	5	− 3	0	2	1	1
784	15.31	65	+ 41	0	+ 1	−15	0	0	8
796	14.93	68	−127	+ 4	+ 3	+ 2	− 3	+ 4	0
804	15.20	70	+ 24	− 2	− 4	− 7	+ 2	− 3	− 2
808	15.30	72	+251	+ 5	+ 6	+ 4	− 5	+ 6	0
816	var.	73	− 25	− 5	− 5	4	+ 3	− 5	+ 4
822	15.09	75	+321	0	+ 2	6	− 4	+ 1	1
829	15.20	77	+ 4	+ 2	− 6	8	0	− 2	4
835	13.23	79	−115	− 5	5	2	+ 3	− 5	2
837	15.42	80	80	+11	− 1	+ 9	0	+ 5	+ 4
840	15.50	80	38	3	+ 1	− 1	− 4	+ 2	− 2
851	15.30	84	20	0	− 6	+ 2	4	− 3	1
852	15.17	85	129	+ 1	+ 1	− 2	7	+ 1	− 4
861	14.86	88	−145	− 2	− 1	+ 2	0	− 2	+ 1
873	15.09	93	+115	+ 2	+ 1	− 5	3	+ 2	− 4
899	15.14	100	+106	0	+ 1	− 4	4	0	− 4
925	15.07	108	−120	+ 8	− 1	+ 4	− 5	+ 4	0
935	14.50	+112	+ 70	+ 3	+ 2	+ 2	0	+ 2	+ 1

TABLE II—*Continued*

No.	Pg. Mag.	x	y	$\mu_{1\alpha}$	$\mu_{2\alpha}$	$\mu_{1\delta}$	$\mu_{2\delta}$	μ_{α}	μ_{δ}
953	15.20	+117″	+ 65″	+ 5	− 2	+ 2	− 1	+ 2	0
978	15.04	132	− 39	− 2	3	− 2	− 5	− 2	− 4
989	15.60	139	− 95	+ 2	6	+ 1	+ 1	− 2	+ 1
1038β	15.07	180	+ 67	+ 7	5	6	− 2	+ 1	2
1042	15.09	182	− 6	− 6	2	4	4	− 4	0
1063	15.08	223	+ 90	+ 2	− 2	6	0	0	+ 3
1079	15.14	255	+350	2	+ 3	5	− 9	+ 2	− 2
1106	15.18	321	−263	4	+ 4	14	+11	4	+12
1113	15.26	+350	+208	+10	− 1	+11	+ 1	+ 4	+ 6

The plate constants a . . . . f and a' . . . . f' were computed in each case from the fifty equations of condition for the fifty comparison stars. These were subsequently substituted in all 112 equations in order to derive $\mu_{1\alpha}$, $\mu_{2\alpha}$, $\mu_{1\delta}$, and $\mu_{2\delta}$. These quantities are therefore proper motions of individual stars with respect to the group of comparison stars. The last two columns of Table II give the mean μ_{α} and μ_{δ} for the two pairs of plates.

The accuracy of the measures is highly satisfactory, the probable error of the motion in each co-ordinate as derived from a comparison of the measures of each pair of plates being only 0.″0023.

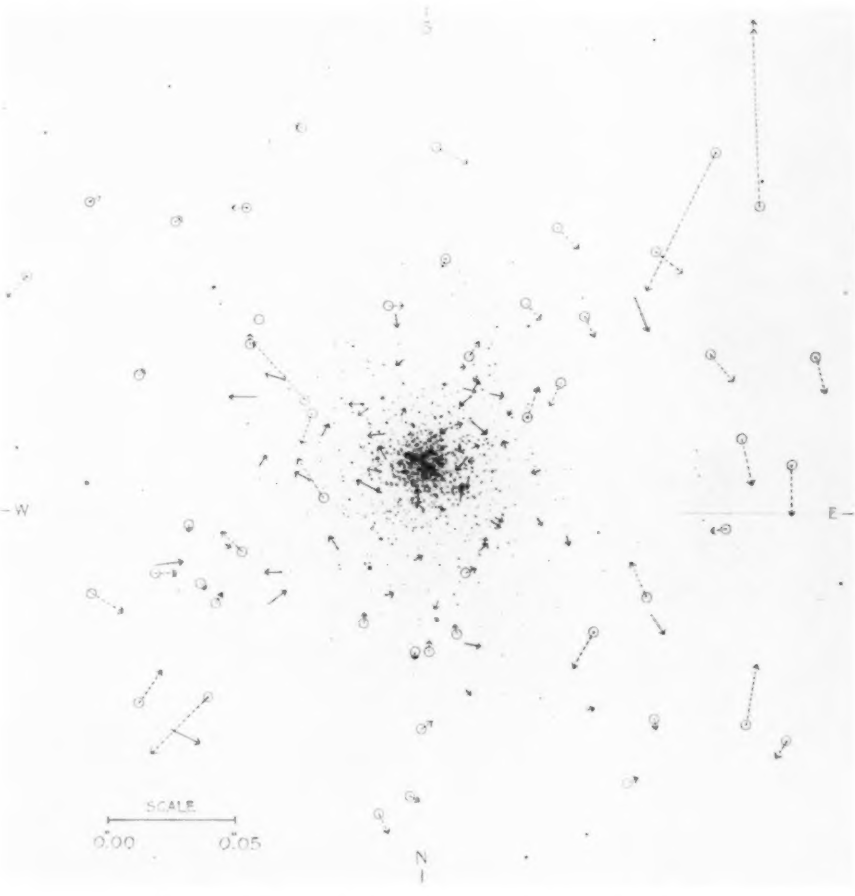
The next step is to derive the mean proper motion of the cluster as a whole. Since the distribution of the stars is nearly symmetrical with respect to the center, the straight mean was taken, giving

$$\mu_{\alpha} = -0.''001 \pm 0.''0004, \qquad \mu_{\delta} = -0.''001 \pm 0.''0003\ ,$$

as the motion of the cluster with respect to the comparison stars. Subtracting these quantities from the last two columns in Table II, we derive the values in the second and third columns of Table III. These differences represent the internal motions and have been plotted in Plate VII. For the comparison stars, which are surrounded by circles, the motions are taken from Table II and are plotted as broken lines. The scale of the motions is given in the lower left-hand corner. The length of the arrows represents the motions in about four thousand years.

For the star s the motion is 50 per cent larger than indicated on the plate. Since its motion is over a tenth of a second, it might better have been omitted, but as it did not show any motion in right

PLATE VII



INTERNAL MOTIONS IN THE GLOBULAR CLUSTER M 13

Star images inclosed in circles represent comparison stars. Images with full-line arrows attached are of stars probably belonging to the cluster, whose motions are relative to the system of comparison stars.

ascension the normal equations in this co-ordinate had already been formed and solved before the displacement in declination was found. Furthermore, as the number of comparison stars is considerable, its influence is small, contributing only a positive displacement in declination to the stars in the southeastern quadrant.

TABLE III

INTERNAL MOTIONS IN M 13

(Given in 0".001 as a unit)

No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{rot.}$	No.	μ_α	μ_δ	$\mu_{rad.}$	$\mu_{rot.}$
2	+11	− 1	−11	+ 3	686	0	+ 2	− 2	0
7	11	+ 5	4	+11	690	− 1	− 1	1	− 1
17	1	− 2	0	− 2	712	− 3	2	− 3	+ 2
25	+ 2	+ 1	− 1	+ 2	737	+ 3	1	+ 3	− 2
46	−11	0	+10	+ 4	775	2	0	+ 1	− 2
49	+ 3	− 4	− 3	− 4	784	1	− 7	− 5	+ 5
59	+ 7	5	− 9	+ 1	796	+ 5	+ 1	+ 1	− 5
70	− 6	0	+ 4	− 4	804	− 2	− 1	− 2	− 1
72	7	2	7	+ 2	808	+ 7	+ 1	+ 3	+ 6
96	− 5	3	+ 4	− 4	816	− 4	5	− 5	− 4
118	+ 3	5	− 2	5	822	+ 2	2	+ 2	+ 2
158	− 3	− 5	− 1	− 6	829	− 1	5	+ 1	− 5
222	5	0	+ 3	+ 4	835	− 4	3	− 5	+ 2
246	− 4	+ 3	+ 1	5	837	+ 6	+ 5	+ 1	− 8
252	+ 1	− 1	− 1	+ 1	840	+ 3	− 1	+ 3	0
299	− 1	0	+ 1	0	851	− 2	0	− 2	0
306	− 5	− 1	+ 4	− 3	852	+ 2	− 3	+ 3	1
310	+ 1	+ 1	− 1	1	861	− 1	+ 2	− 2	− 1
311	−10	− 5	+ 4	−10	873	+ 3	− 3	1	+ 4
313	1	+ 1	1	0	899	1	− 3	− 2	+ 2
323	7	+ 1	5	+ 5	925	5	+ 1	+ 3	− 4
325	− 3	− 5	+ 3	− 5	935	3	2	4	0
326	+ 3	− 1	− 1	+ 3	953	+ 3	+ 1	+ 3	+ 1
363	1	+ 5	− 5	0	978	− 1	− 3	0	+ 3
406	+ 1	− 3	+ 3	− 2	989	− 1	+ 2	− 2	− 1
418	− 3	+ 2	− 2	+ 3	1038a	+ 2	3	+ 3	2
500	+ 3	− 2	2	+ 3	1042	− 3	1	− 3	1
523	0	5	5	− 1	1063	+ 1	+ 4	+ 2	− 3
559	− 3	3	3	− 3	1079	3	− 1	+ 1	+ 3
641	+ 1	− 1	− 1	+ 1	1106	5	+13	− 5	−13
653	− 1	+ 3	+ 3	− 1	1113	+ 5	+ 7	+ 8	− 3

A glance at the plate shows that the internal motions must be very small, and a discussion of the data corroborates this fact. If we accept for a moment that the motions of the cluster stars are due to accidental errors only, we find a probable error in each coordinate of 0".0026. It is true that this is slightly larger than the value stated above, but it is well known that the internal probable

error is always somewhat, and sometimes considerably, smaller than the external probable error. The difference between the two values given indicates that the internal motions of the members of the cluster can scarcely exceed 0."001 in each co-ordinate.

For completeness the motions were further resolved into two components, one radial and one perpendicular thereto. These are given in the last two columns of Table III. The first component is one that might a priori be expected from a contraction or dispersion of the cluster; the result, however, is that the mean radial component is exactly zero.

There is little reason to anticipate a component at right angles to the radius, except that such a component has been shown to exist in the measures of the spiral nebulae. In M 13 this component comes out 0."0005±0."0004, in the direction N-W-S-E.

We may thus conclude that the internal motions in the cluster are of the order of 0."001 or less. A considerably longer interval will therefore be required to detect any possible motion.

Although these results are negative, they are, nevertheless, of importance for two reasons. First, they are a beautiful confirmation of Shapley's distances, because with the parallax derived by him, 0."00009, even a fairly large velocity would still give an extremely small proper motion; and second, because the results obtained for M 13 show again that the internal motions found in several spirals cannot be due either to the 60-inch reflector or to the measuring instrument. The only possibility that the displacements found in the spirals are not real motions would seem to lie in a systematic difference along the spiral arms between the old and the new plates. This possibility is extremely doubtful, because in that case the effect would hardly be proportional to the time interval, as was found in several of the spirals for which pairs of plates with different time intervals were available. Moreover, such displacements would be considerably less in the case of an object with starlike points, such as M 33, than in some of the others. Finally, we should expect such a systematic error to be larger near the center than near the edge of a spiral, while the measures show just the reverse in all cases.

MOUNT WILSON OBSERVATORY
August 20, 1924